Contagion and Risk Sharing on the Inter-bank Market

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• Interbank loans are conducted directly between pairs of banks

• If a bank fails its creditors may not recover their funds

• Counterparty risk -> Systemic risk
Questions

What is the effect of the structure of the interbank market on the likelihood of contagion?

How this varies with the type and degree of shock?

How financial regulation may be used to mitigate these effects?

Is there an optimal interbank market structure?
Previous Work

• Markets as risk sharing

• Markets as contagion spreading

• Structure
  – Iori et al. (2008) – Italian
  – Muller (2006)* – Swiss
  – Upper and Worms (2004)* – German
  – Boss et al. (2004) – Austrian
  – Angelini et al (1996) – Italian
  – Humphrey (1986) *- USA
Previous models

• Several papers have attempted to understand the effect of market structure
  – Iori (2006)
  – Battiston et al (2009)
  – Gai and Kapadia (2010)
  – Georg (2012)
  – (and others)
Model

- Discrete time, partial equilibrium model of the interaction of:
  - Financial Sector
    - N Banks
  - Non-financial sector
    - M Lenders
    - Q Borrowers
- All agents uniformly distributed on a unit circle
• Depositors – each time step
  – Place all of their deposits, \( D_j \), in a single bank
    \[
    \arg \max_{i \in N} D_j (r_i^{\text{deposit}} - g(i, j))
    \]
  – Where \( g(i,j) \) is the distance between \( i \) and \( j \) on the circle

• Borrowers
  – Receive a single investment opportunity which requires \( \tau \) and pays \( \mu \) after two periods with probability \( \theta \)
  – Approach a single bank to fund that investment
    \[
    \arg \max_{i \in N} \tau \theta_{I_{j,t}} (\mu - (1 + r_i^{\text{loan}})^2) - g(i, j)
    \]
• **Banks**

– Each has a balance sheet consisting of:
  - Interbank loans $I_i$
  - Loans to non-financial entities $L_i$
  - Deposits $D_i$
  - Equity $E_i$
  - Reserves $R_i$

– Sets too interest rates
  - Loan rate $r_{i}^{\text{loan}}$
  - Deposit Rate $r_{i}^{\text{deposit}}$
Banks

– Maximise expected return

\[
E(r_i) = \left( \sum_{k_i=1}^{K_i} (1 + r_{i,loan}^{k_i}) \theta_{k_i} \right) + (1 + r_{interbank}^{i}) I_i \theta_{interbank}^{i} - r_{i,deposit}^{i} D_i
\]

- Funded Loan Request
- Interbank Lending and Borrowing
- Deposits

– Key aspect is the selection of \( K_i \) – the set of funded loan requests
Constraints:

• Balance sheet must balance

\[ E_i + D_i = L_i + R_i + I_i \]

• Deposits are equal to the value paid in by households

• The value of loans is the sum of investments funded

• Reserves requirement

\[ R_i \geq \alpha_i D_i \]

• Capital requirement

\[ E_i \geq \beta_i (L_i + \max(I_i, 0)) \]
• Interbank Market
  – Interest rate is determined endogenously
  – Rate at which supply of interbank funds equals demand

• Given an interbank rate a particular bank maximise their expected return and so lend/borrow a specific amount.

• Over the population of banks this may lead to excess supply or demand

• Equilibrium rate is found numerically
OTC market – direct connection between lenders and borrowers

Connections determined randomly
– Consider each borrower in turn
  • Borrows form each lender with probability $\lambda$ (probability of connection)
  • Loan proportional to requested funds
  • If insufficient funds are available to meet borrowers demand more borrowers add added in decreasing order of size
If a bank has negative equity or is unable to repay it interbank loans it goes bankrupt
   – 100% of available funds are used to repay loans

Dividends
   – At the end of each time step banks pay dividends proportional to their equity such that equity over time is constant.
Bank parameters
- Reserve Ratio (Reserve requirement)
- Equity Ratio (Capital requirement)
- Lending Rate
- Deposit Rate
- Probability of repayment of interbank loans

Initially randomly assigned values

Each time step two banks are randomly chosen
- Poorer banks parameters are replaced by perturbed copies of richer banks parameters
- Perturbation U(0,0.001)
# Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
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<tr>
<td>$\beta$</td>
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<tr>
<td>$\alpha$</td>
<td>Reserve Requirement</td>
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<tr>
<td>$N$</td>
<td>Banks</td>
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<td>$M$</td>
<td>Households</td>
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<td>$\mu$</td>
<td>Project Payoff</td>
<td>0.15</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Project Success Probability</td>
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<td>$\tau$</td>
<td>Project Size</td>
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Model simulated for 10000 time steps before statistics generated. 25000 repetitions

T-test that parameters no longer change
# Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>Value</th>
<th>Std Dev.</th>
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<tbody>
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<td>Deposits</td>
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<td>IB Loans</td>
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<td>Equity</td>
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<td>Unused Capital</td>
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<td>Cash Reserves</td>
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<td>Other Liabilities</td>
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<td>Loan Rate</td>
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<td>Dividend Rate</td>
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<td>Borrower Equity</td>
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Individual bankruptcy

<table>
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<tr>
<th>$\lambda$</th>
<th>Interbank Connections</th>
<th>Number of bankruptcies</th>
<th>Contagion %</th>
<th>Size given contagion</th>
<th>Largest Shock</th>
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<td>0.22</td>
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<td>16.79</td>
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</table>

Probability – Angelini et al. (1996) and Boss et al. (2004)
Systemic Shock

– Change probability of project success

\[ \theta^{\text{shock}} = 0.84 \]

\[ \theta^{\text{shock}} = 0.86 \]

\[ \theta^{\text{shock}} = 0.88 \]

\[ \theta^{\text{shock}} = 0.9 \]

\[ \theta^{\text{shock}} = 0.92 \]

\[ \theta^{\text{shock}} = 0.94 \]
Systemic component is significant:
- Giesecke and Weber (2006) and Brusco et al. (2007)

Interbank rate increases
- particularly for unconnected markets

Interbank market confidence heavily reduced
- Brusco & Castiglionesi (2007)

Loans and deposits reduce proportionate to size of the shock

Effects of bankruptcy continue into the future
- Intermediately connected markets particularly badly effected
• Welfare effect – cost of bankruptcies
Market Confidence

• During the financial crisis confidence in the market evaporated

Reduce probability of being repaid by

$$\kappa B_t$$

Where kappa is optimised for each bank and $$B_t$$ is the number of bankruptcies this period
Credit worthiness

• Banks are more willing to lend to those banks who are more likely to repay
  – In this case those with more equity

• Rate at which banks may borrow at is increased for each bank by
  \[ |N(0,1/E\downarrow j)|\% \]
## Credit and Market Confidence

<table>
<thead>
<tr>
<th>Market Confidence</th>
<th>Number</th>
<th>Max Size of Failure</th>
<th>Credit Worthiness</th>
<th>Number</th>
<th>Max Size of Failure</th>
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<td>0.2</td>
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<td>0.3</td>
<td>1.1</td>
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<td>0.78**</td>
<td>22.3**</td>
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<td>0.4</td>
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<td>0.7</td>
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<td>1</td>
<td>0.57**</td>
<td>41.9**</td>
<td>0.16</td>
<td>23.16</td>
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</tr>
</tbody>
</table>
Conclusions

The market does a good job of replicating stylised facts of real interbank markets, despite its simplicity.

For small shocks, more connected markets are less likely to suffer from contagion but when they do it may be more severe.

The relationship between systemic shocks, the structure of the interbank market and contagion is complex.

No optimal Inter bank market structure.
Conditioning on credit worthiness leads to increased stability.

Conditioning on Market confidence has the opposite effect.