Quantifying collective animal behaviour

David J. T. Sumpter
Mathematics Department, Uppsala

Work with: Ashley Ward, Teddy Herbert-Read and Jens Krause (Berlin/Sydney)
Daniel Strömbom, Arianna Bottinelli, Richard Mann & Andrea Perna (Uppsala)

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STARFLAG project: http://angel.elte.hu/starling/index.html
Slime mould and Ant Networks

Tero et al, 2010, *Science*


Latty et al, 2011, *Royal Society Interface*
Collective behaviour
What questions do we ask?
Collective behaviour
What questions do we ask?

• Group decision-making.

• Information transfer.

• Collective motion.
Collective behaviour
What questions do we ask?

• Group decision-making.

• Information transfer.

• Collective motion.
Predator detection

Ward et al. (2011), Proceedings of the National Academy of Sciences
predator to left

Ward et al. (2011), Proceedings of the National Academy of Sciences
Predator avoidance increases with group size

\[ 1 - \frac{1}{2} (1 - 0.11)^n \]

Ward et al. (2011), *Proceedings of the National Academy of Sciences*
Time taken decreases with group size

Mean (±SE) time (s) spent by focal fish in each zone

Ward et al. (2011), *Proceedings of the National Academy of Sciences*
Quorum responses

Collective behaviour
What questions do we ask?

- Group decision-making.
- Information transfer.
- Collective motion.
Look up!

Gallup, Hale, Sumpter, Garnier, Krebs, Kacelnik & Couzin, submitted
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Model: Low flow

Data: Low flow

Model: High flow

Data: High flow
Collective behaviour
What questions do we ask?

• Group decision-making.

• Information transfer.

• Collective motion.
Buhl et al. (2006), Science
Yates et al. (2009), PNAS
Buhl et al. (2006), Science
Buhl et al. (2006), Science
1D self-propelled particles

7 particles

Position:
\[ x_i(t + 1) = x_i(t) + v_0 u_i(t) \]

Velocity:
\[ u_i(t + 1) = \alpha u_i(t) + (1 - \alpha) G \left[ \left\langle u_j(t) \right\rangle_{|i-j|<r} \right] + \xi_i, \]

Buhl et al. (2006), Science
1D self-propelled particles

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Buhl et al. (2006), Science
Model vs Data

Buhl et al. (2006), Science
Model vs Data

Buhl et al. (2006), Science
Self-propelled particle models

\[
x_i(t + 1) = x_i(t) + v_0 u_i(t + 1)
\]

\[
u_i(t + 1) = f(u_i(t), \{ (x_j(t), u_j(t)) : j \text{ is a neighbour} \}, e)
\]

Too many models, not enough data

• Thousands of (possibly 10,000) papers on SPP models. Many of which generate realistic looking schooling behaviour.
• Some ‘universal’ properties established of these models (Vicsek & Zafiris, 2010).
• Few studies which identify the actual rules of interaction.
\[ P(\text{direction change} \mid s) = \frac{1}{1 + \exp(-s)} \]

- **Independent (N)**
  \[ s = q \]

- **Global Interaction (MF)**
  \[ s = q + \lambda \cdot N \]

- **Nearest Neighbours (T)**
  \[ s = q + \lambda \cdot N_\mathcal{K} \]
  set \( \mathcal{K} \) of nearest-neighbours

- **Interaction Radius (S1-4)**
  \[ s = q + \lambda \cdot N_\mathcal{R}^- + \lambda \cdot N_\mathcal{R}^+ \]
  zone-of-interaction, \( \mathcal{R} \)

\[ P(\text{direction change} \mid s) = \frac{1}{1 + \exp(-s)} \]

Independent (N)
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- **Interaction Radius (S1-4)**
  \[s = q + \lambda_{{-}} N_{{R}{{-}}} + \lambda_{{+}} N_{{R}{{+}}}\]

  zone-of-interaction, \(\mathcal{R}\)

\[ P(\text{direction change} \mid s) = \frac{1}{1 + \exp(-s)} \]

Independent (N)

\[ s = q \]

Global Interaction (MF)

\[ s = q + \lambda N \]

Nearest Neighbours (T)

\[ s = q + \lambda N_K \]

set \( K \) of nearest-neighbours

Interaction Radius (S1-4)

\[ s = q + \lambda N_{R^-} + \lambda N_{R^+} \]

zone-of-interaction, \( R \)

\[ P(\text{direction change} \mid s) = \frac{1}{1 + \exp(-s)} \]

- Independent (N): \( s = q \)
- Global Interaction (MF): \( s = q + \lambda_- N_- \)
- Nearest Neighbours (T): \( s = q + \lambda_- N_K_- \)
- Interaction Radius (S1-4): \( s = q + \lambda_- N_{R_-} + \lambda_+ N_{R_+} \)

\[ P(\text{direction change} \mid s) = \frac{1}{1 + \exp(-s)} \]

Independent (N)
\[ s = q \]

Global Interaction (MF)
\[ s = q + \lambda N_\cdot \]

Nearest Neighbours (T)
\[ s = q + \lambda N_{K_\cdot} \]
set \( K \) of nearest-neighbours

Interaction Radius (S1-4)
\[ s = q + \lambda N_{R_\cdot} + \lambda N_{R_+} \]
zone-of-interaction, \( R \)

\[ P(D|M) = \int P(D|\phi, M)P(\phi)d\phi \]

**Bayes factor test**

\[ BF = \frac{P(D|M_1)}{P(D|M_2)} \]

Fine-scale model selection

Forward interaction with those moving in opposite direction.

Interaction radius equal to half a prawn body length.

Fine-scale model selection

Global-scale fit

Data

Model
Turning example
\[ P(\text{direction change} \mid s) = \frac{1}{1 + \exp(-s)} \]

Interaction Radius with Memory (D1-4)

\[ s_t = ds_{t-1} + (1 - d)[q + \lambda_- N_{R-}^{t-1} + \lambda_+ N_{R+}^{t-1}] + \lambda_- N_{R-}^t + \lambda_+ N_{R+}^t \]

Fine-scale model selection

Global-scale fit

Data

Model D3
Herbert-Read et al. (2011), PNAS
## Rules of motion

Herbert-Read et al. (2011), *PNAS*

<table>
<thead>
<tr>
<th></th>
<th>-135</th>
<th>-90</th>
<th>-45</th>
<th>0</th>
<th>45</th>
<th>90</th>
<th>135</th>
<th>180</th>
</tr>
</thead>
</table>
Catch up those in front.
Wait for those behind.
Turn towards your neighbours
Attraction is more important than alignment
Fish swim parallel to sides of tank
Response = Past + Wall + Neighbour 1 + Neighbour 2 + Neighbour 3

Herbert-Read et al. (2011), PNAS
Herbert-Read et al. (2011), *PNAS*
Herbert-Read et al. (2011), *PNAS*

Primarily Nearest Neighbour

Acceleration Response

Repulsion/Attraction

Leader/Follower
Primarily Nearest Neighbour

Acceleration Response

Repulsion/Attraction

Leader/Follower

Herbert-Read et al. (2011), PNAS
Herbert-Read et al. (2011), *PNAS*

- Primarily Nearest Neighbour
- Acceleration Response
- Repulsion/Attraction
- Leader/Follower
Primarily Nearest Neighbour

Acceleration Response

Repulsion/Attraction

Leader/Follower

Herbert-Read et al. (2011), *PNAS*
Perna et al. (submitted)
Simple model

\[ P_{\text{stop}} = \lambda_1 + \lambda_2 \rho_{\text{stopped}} + \lambda_3 \frac{n_{\text{claps}}}{\max n_{\text{claps}}} + \lambda_4 \rho_{\text{neighbours stopped}} \]
Bayes factor test

\[ P_{\text{stop}} = \lambda_1 + \lambda_2 \rho_{\text{stopped}} + \lambda_3 \frac{n_{\text{claps}}}{\max n_{\text{claps}}} + \lambda_4 \rho_{\text{neighbours stopped}} \]
Webpage:

http://www.math.uu.se/~david/

Just now Postdoc position available to work with Jens Krause and myself on human collective behaviour.

http://www2.personalavd.uu.se/ledigaplatser/2450postDOC.html

www.collective-behavior.com