ABSTRACT

In this paper we present a new interdisciplinary perspective on regional interaction patterns in archaeological contexts. It combines insights from graph theory, social network analysis and statistical physics to treat the interactions between sites in geographical space in terms of a network which minimises an associated Hamiltonian. To explore the various issues involved a case study from a heterogeneous physical environment is chosen, the archipelago environment of the southern Aegean, in particular the rich dataset of the Aegean Bronze Age. Our findings are of broader relevance for the study of interaction networks, as the use of statistical physics in this fashion represents a novel application in social science contexts.

I. RECONCILING PHYSICAL AND RELATIONAL SPACE

One might imagine that the spatial relationships between entities, across a range of scales, would constitute a fundamental component of any archaeological analysis. However, space has received a surprisingly uneven treatment within the discipline, with spatial analysis only really coming to the fore in the 1960s and 70s, through the influence of the ‘New Geography’ (Haggett 1965; Chorley and Haggett 1967), channelled into archaeology principally through David Clarke (Clarke 1968; 1977). Clarke describes three levels of resolution in spatial archaeology: the micro level, the semi-micro level and the macro level, the last of these being relationships between sites, the level at which models from economics and geography are most relevant (Clarke 1977, 13). In this paper we focus on this, the macro level, examining interactions between sites across a heterogeneous physical space – the archipelago environment of the southern Aegean.

Yet the state of play with regard to spatial analysis has, needless to say, moved on considerably since the days of Haggett and Clarke, in both geography and archaeology respectively. In geography a critique emerged relatively rapidly, with spatial analysis accused of spatial fetishism: that the social relations occurring in space were largely determined by physical space. Rather than asking what space is, Harvey (1973) asked how different human practices actively create space. This ultimately came to fruition in the 1990s in ideas of ‘relational space’ (Harvey 1996), and notions of ‘space-times’ as being constituted through human action and interaction (Thrift 1996; Hetherington 1997). In archaeology, on the other hand, the geometric approach to space took root more firmly, with techniques of locational analysis, such as Christaller’s central place theory, continuing to be used into the 1980s (e.g. Wagstaff 1987, although some papers in this volume do show some hints of unease with geometric determinism). By the 1990s, however, the post-processual critique has kicked in and approaches to space have been ‘relationalised’, largely through the influence of phenomenology on the study of the landscape (Bender 1993; Tilley 1994; Knapp and Ashmore 1999). Moreover, this shift in emphasis, toward viewing space as constituted by human action rather than as given, is
accompanied by the use of the term ‘landscape’ in favour of ‘space’; a movement from space to place (Hirsch 1995).

In both disciplines, however, the move towards relational conceptions of space and away from geometric determinism has arguably created a dualism between relational and physical space. It is our aim in this paper to develop a methodology that can go some way toward bridging the gap that has opened up between them (cf. Hillier 2005). What is required is an approach that incorporates the fundamental notion that humans create space through social practices, while also acknowledging that physical parameters are not entirely redundant in this process. One of the misconceptions hindering this rapprochement has been that spatial analysis is bound to Euclidean geometry; however, recent advances in complexity science, and in the study of complex networks in particular, give the lie to this idea (e.g. Batty 2005, on networks in geography). What these advances allow for is the evolution of spatial dynamics from the bottom-up, in ways seemingly unimaginable to central place theory or core-periphery models.

While complexity science has certainly had a major influence on our approach, we believe that some of the problems with spatial analysis can actually be worked through at a more basic level. A fundamental problem is one of emphasis – in much spatial analysis, even in the more sophisticated forms of Geographical Information Systems (GIS), interactions between points are seen as secondary to the existence of those points. It is what Batty has described as ‘the geography of locations, not relations’ (Batty 2005, 149). The equivalent to this in the archaeological analysis of regional systems is that the sites are thought to emerge and gain their character on largely local grounds, and any interactions with other communities in the region follow on from that. The connections between sites are simply drawn as lines, without weight or direction. Such ‘site-centrism’ makes it difficult to entertain the thought that site interactions might themselves contribute to the size and status of the sites in question.

How, then, might we turn the tables, and treat interactions as primary and sites as secondary? How can we achieve what we might dub, borrowing from Batty, an ‘archaeology of relations’? Surely such a move is justified, not least in environments with patchy resources in which community self-sufficiency seems particularly improbable? Let us now try to answer these questions through a case study: The Aegean Bronze age.

II. THE AEGEAN BRONZE AGE AS A CASE STUDY

The central idea is to use the technology of networks to represent the archaeology of relations. For the best case studies we would like physical impediments, such as mountain ranges or deserts, together with technological limitations to severely limit the possible interactions in society. Island archaeology provides an ideal laboratory. The dominant interactions are controlled by a single mechanism: the ability to cross open sea. The simpler technologies of the past will further limit the possible interactions both within a region and without. These limitations often contribute to a slow pace of change and so a quasi-static equilibrium, a useful feature for model building. Islands also provide a natural network structure. Thus it is not surprising that both Oceania and the Aegean have featured in network-based studies (Terrell 1977; Irwin 1983; Davis 1982; Hage and Harary 1991, 1996; Broodbank 2000).

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1 We can be much more certain that with only rowing technology, a daily range of 10km is reasonable, while by sail 100km is not impossible.
In particular the Bronze Age Aegean is an ideal laboratory for network analysis - its mountainous land masses isolate what are primarily coastal communities, the natural vertices, linked largely by sea communications (see Figure 4). It is also relatively self-contained in space and time\textsuperscript{2}. Earlier work on island archaeology in the Aegean (Davis 1982; Broodbank 2000) focused on the EBA (Early Bronze Age, up to c. 2000 BC) where the rowing technology and limited sizes of sites required only simple network methods for their analysis.

The Aegean Bronze age already provides an excellent example of a ‘turning of the tables’, putting interactions ahead of vertices. Broodbank studies the Cyclades, the central group of Aegean islands around Ios (see Figure 4), in the pre-sail era which finished around 2000 BC (Broodbank 2000). It is the only systematic attempt thus far, for any period of the prehistoric Aegean to explain the growth of certain sites (in the Cyclades) in terms of their interactions. This approach was perhaps encouraged by the fact that some important Early Cycladic sites are very hard to explain in terms of local resources, occurring on small rocky islands with limited agricultural or mineral resources. It thus seemed likely that location had a substantial role to play in a site’s importance. Broodbank compared the results of his networks, simple graphs in the mathematical sense, with the archaeological. Of the five major sites in his region, three were ‘central’ in his networks.

In terms of the methodology Broodbank employs, his use of graph theory is instructive. A mathematical technique of this kind, however simple, may seem initially inconsistent with the ‘humanistic’ tone of Broodbank’s work. Yet his work marks an important step in bridging the gap between complex spatial modelling methods with limited contact with social parameters and, on the other hand, humanistic discussions of socio-spatial patterning without mathematical analysis. Moreover, it allows him to demonstrate graphically that, even in simple interaction networks, interactions can contribute just as much as the sites to the overall character of the network. It shows us that a rapprochement between physical and relational space is not impossible, and that it can be achieved to some extent without the most recent advances in complex networks. This is not to say that such advances would not allow us to go further still with this rapprochement, and achieve a more secure bridging of relational and physical space.

III. MODEL BUILDING

Our primary goal is to study the MBA (Middle Bronze Age, c. 2000- 1200 BC). This is well bounded in time as the record shows significant gaps at the boundaries of this period. Further the sail appears c. 2000 BC which facilitates new levels of inter-regional interaction and the exchange networks of the EBA metamorphose into affiliation networks.

This throws into doubt one of the fundamental assumptions of PPA (Proximal Point Analysis) used by Broodbank and some of the earlier studies. There sites are linked to a fixed number of nearest neighbours. While this might make excellent sense for rowing technology, with the sail the distances travelled could easily increase by an order of magnitude. Even if such long trips were still not the norm, sail technology may make them just significant enough that they form the basis of important if weak links in the sense of Granovetter (Granovetter 1973, 1982).

\textsuperscript{2} Connections with regions beyond the Aegean, in particular with Egypt, remain controversial in this period. A longer term goal would be to include such ‘external sources’, to use the scientific terminology, to provide a network perspective for this debate.
The region in this period also sees a much discussed process occurring, known as ‘Minoanisation’, as hierarchical structures emerge with the Minoan culture. One question we want to answer is why some sites, like Knossos on Crete, grew to be so large and influential. The size of such sites is usually explained in local terms of surplus and growth, with these local conditions then enabling exchange with other sites. We are interested in reversing this equation, exploring the possibility that some characteristics of the larger interaction networks contributed to the growth of such sites.

This all means our network models need to be more complex than those in the literature: MBA communities (vertices) need to show large variations in size and character, open sea links (edges) are sensitive to currents and prevailing winds. Against these challenges, there is far more archaeological material for the MBA Aegean against which we can compare our models and it means our models will have more widespread applicability.

The overall approach we have chosen to adopt portrays a network, with its complicated constraints and interactions, as explicable in terms of a Hamiltonian $H$. An optimal configuration is one which has a low energy. We use a Boltzmann distribution and create one network $G$ with probability $p(G) \propto \exp (-\beta H(G))$ where $\beta$ is a constant. We can look at typical (most likely) configurations or we can also look at averages over an ensemble of good solutions. Under some circumstances this can be equivalent to the time average of a single representation of the system. The implicit statistical fluctuations in the networks we construct reflect the normal variations in a real world system.

Statistical physics provides general theorems explaining when and why the Boltzmann distribution emerges, as it often does, from systems of many smaller parts. We are therefore exploiting basic counting arguments for a large number of interacting objects when we follow this approach. Thus our method makes few fundamental assumptions and it is not too prescriptive. At the same time by making contact with statistical physics, we can draw on over a century of work, both in terms of concepts and in terms of practical algorithms.

All the uncertainties about how to model society are pushed into our choice of ‘energy function’, or Hamiltonian $H$. There is a general issue here, alluded to above, which applies to any modelling of social, cultural or economic phenomena by algebraic methods. Such methods can seem naïve, while simultaneously being over-prescriptive in that definite (but potentially arbitrary) functional forms have to be chosen so that calculations can be performed. As to the former, the reader must decide, but the latter is less a problem than one might think. One resolution to being over-prescriptive is the notion of a universality class. By this is meant that, rather than try to prescribe a ‘fuzzy’ function to accommodate our uncertainty, we can hope for a family of ‘crisp’ functions that, provided we ask the right questions, will all give us the ‘same’ answer. The notion of topological congruence, taken from population biology, is most helpful. Functions which can be deformed into one another by stretching and squeezing are topologically congruent. Although we consider a specific function, we expect similar general results from different functional forms in a family of functions, as long as they are congruent. This is one way to characterise robustness, which is essential if we are to believe that our conclusions are realised by realistic systems.

Within this framework our primary principle is to work with the fewest number of arbitrary parameters and the simplest functional forms that are required on general
grounds, taking the details of the archaeological record into account step by step. Experience suggests that the non-linearities inherent in even the simplest models will produce complex behaviour of the sort we observe in practice. We will also demand certain transformation properties which we will describe below.

**Sites:** Each site $i$ is given a physical location, a fixed characteristic size $S_i$ and a variable occupation fraction $v_i$ to be determined. One possible representation is that the active population at a site is $(S_i, v_i)$ with $S_i$ setting the maximum self-sustainable population at a site. Small rocky islands will have small $S_i$ yet they might have a large population, $v_i \gg 1$, if they play a pivotal role in the global network. We will denote the total number of sites by $N$. We have tested using a list of 34 known MBA sites shown in Figure 4 and assigned all equal relative sizes $S_i=1$. We plan to look at other possibilities later.

**Edges:** We associate an edge variable $e_{ij}$ to each link between sites $i$ and $j$. One interpretation is that $e_{ij}$ represents the trade from site $i$ to site $j$ and may not equal $e_{ji}$. We also define an effective distance $d_{ij}$ from site $i$ to site $j$ which here is just the distance between the two sites. Later we will modify $d_{ij}$ to take account of difference between land and sea transportation, prevailing winds and currents and so forth.

**Parameters:** The parameters that control the contours of the landscape are measures of site independence or self-sufficiency, and constraints on population size, etc. Thus, for example, as populations grow or total trade volume increases, the optimal network (lowest energy configuration) changes.

Volatility in the system, such as short periods of drought, can be accommodated through the inverse ‘temperature’ $\beta$ in the Boltzmann distribution, whereby high volatility is ‘hot’, low volatility ‘cold’.

**Transformation Properties:** To further constrain $H$ we demand that it behaves appropriately under special transformations. One such principle is the symmetry of the form of $H$ under the interchange of labels of any two sites. That is, every site is governed by the same type of interactions as any other. This does not mean that every site is identical; we break this homogeneity when we incorporate different resources, $S_i$, and unequal distances $d_{ij}$ between sites.

A further attribute that we may wish our model to have is what we term ‘block renormalisation’. That is if we were to split a single site into two sites close together of the same total size, then we wish the energy of the configuration to be invariant. In this way the precise determination of what was the centre of any one site should be unimportant. If this is imposed the sites in Figure 4 can then be understood as ‘supersites’, aggregates of the smaller island communities that, in the EBA, were individually the vertices of that network. Similarly, the edges or links are ‘superedges’ or ‘superlinks’ that, for example, describe the interaction between island and island, aggregates of the many individual links between site and site. In this way robustness is built into the model. We stress that, although this minimises the effects of our ignorance of the detailed archaeological record, this in itself is no guarantor of its correctness. Nonetheless, this makes it a convenient starting point in model making.

**Hamiltonian:** The example we have proposed in our initial proof-of-concept studies that embodies the above is
\[ H = -\kappa \sum_i S_i v_i (1 - v_i) - \lambda \sum_{i,j} V(d_{ij}/D) (S_i v_i) (S_j v_j) \]
\[ + j \sum_i S_i v_i + \mu \sum_{i,j} S_i v_i e_{ij} \]

The sums are over the different sites or over all pairs of supersites, labelled by \( i \) or \( j \) (henceforth we drop the prefix 'super' both for sites and links). The first term proportional to a constant \( \kappa \) controls the size of sites as if there were no outside contacts. It is the logistic map as used for simple models of population dynamics. Sites have negative energy for \( 0 < v_i < 1 \), while for values larger than 1 the cost is positive. Note this term is invariant if we split a site into two by dividing \( S_i \) between the two new sites but keep the occupation fraction \( v_i \) the same for both new sites – our block renormalisation principle.

The second term allows for interactions, 'trade'. It is proportional to the total 'populations' at both ends of a link \((S_i v_i)\) and to an edge weight variable \( e_{ij} \). This permits block renormalisation since, although the number of possible edges involved also doubles, the total energy remains the same provided we ignore any new edge between the two new sites. For such models it is advantageous, in cultural exchange, or trade, for both a site and its exchange partner to have large resources. We realise that the cultural exchange/transmission that we are considering here is by no means simply economic but, in contemporary economic parlance, we would say that this model embodies the advantages of a large consumer market and producer power.

This term is controlled by a constant \( \lambda \) and by a potential term \( V \) which we choose to vanish for small distances, at the scale at which site bifurcation entails no cost, and to be \([ (d_{ij}/D)^4 + 1 ]^{-1/4} \) at larger scales, falling off to zero on a distance scale \( D \), where \( d_{ij} \) is the effective distance between site \( i \) and site \( j \), \( D \), the distance of a typical day's journey, is set by the sailing technology to be between 50 and 100km. All other things being equal, increasing \( \lambda \) increases the importance of inter-site interaction, whereas increasing \( \kappa \) increases the importance of single site behaviour. Thus, in the EBA Cyclades, when islands are not self-sufficient, \((\lambda/\kappa)\) is relatively large.

The final terms enable us to impose constraints on population size, total trading links (and/or journeys made). Increasing \( j \) effectively corresponds to reducing population, and increasing \( \mu \) reduces exchange. In the language of statistical systems, \( j \) and \( \mu \) are 'chemical potentials' and we are working with a Grand Canonical ensemble.

**Summary:** Given the sizes \( S_i \) and the separations of the sites, the aim is to find the network configuration \( G \), the values of the occupation fractions \( v_i \) and edges \( e_{ij} \), that makes \( H \) as small as possible for fixed values of \( \kappa, \lambda, j, \mu \) and \( D \). If we want some volatility, we keep the temperature non-zero though \( \beta \) and then we can consider ensemble averages.

**IV. ANALYTIC (MEAN-FIELD) SOLUTIONS**

Before attempting any numerical modelling with the real island parameters, it is useful to see some of the behaviour that might arise, using simple analytic approximations for an idealised network of sites. We work at zero temperature and make a mean field approximation in which we replace every value of \( v_i \) and every value of \( e_{ij} \) in \( H[v_i, e_{ij}] \) by their average values \( v \) and \( e \) respectively. We then look for minima of \( H(v, e) \) a two-dimensional energy landscape subject to constraints that
\(0 \leq v \leq 1\) and that \(0 \leq e \leq 1\). In some cases the lowest values will be at one of the boundaries and indeed the energy landscape will force the system to move to extreme values in one or both parameters.

As we have suggested, increasing \(\lambda\) increases the importance of inter-site interaction, whereas increasing \(\kappa\) increases the importance of single site behaviour. If \((\lambda/\kappa)\) is relatively small the latter effect may overwhelm and we expect a stable energy minimum. That is the advantage of being close to the optimal population \(v=0.5\) is too great for trade to matter very much. The plot in Figure 1 for small \(\lambda\) does indeed show a valley near this value. On the other hand, when islands are not self-sufficient and \((\lambda/\kappa)\) is relatively large the latter effect may not be enough to inhibit runaway growth as trade brings benefits that outweigh local overpopulation effects and this is seen in Figure 2. However, in this situation we have a saddle point and there are two possible outcomes: the runaway growth or collapse of the system. That is it may be better to reduce the population to reduce the penalty of having large populations and suffer the loss of advantageous trade. Iterating this brings us to collapse. Which wins depends on which side of the saddle point leads to the lower valley bottom. In general this will not be a blanket collapse. There will be a mixture of valleys and cols in this multidimensional landscape and not all of the latter will be traversed in the direction of local collapse. Nonetheless, this shows the ease with which many sites in the network can either disappear \((v_i = 0)\) or cease to communicate \((e_{ij} = 0)\).

Roughly, provided \(\lambda\) is large enough then, as \(\lambda\) increases from zero for fixed \(\kappa\), there is a monotonic growth in average site exploitation from under-exploitation to full exploitation. Provided \(\lambda\) is large enough then, if \(\lambda\) is held fixed and we increase \(\kappa\), all sites undergo medium exploitation as trading links become unimportant. The major difference occurs when \(\lambda\) (trading strength) decreases for small fixed \(\kappa\) (low self-sufficiency). Then, for only a small reduction in trading strength, exploitation of resources can collapse from full exploitation to no exploitation which, naively, we might infer as site abandonment. This is shown in Figure 3 in which we shown the mean field average of \(v\) for varying \(\lambda\) and \(j\), for fixed \(\kappa\) and \(\mu\).

In this regard we note the following observation by Broodbank et al. (in press):

“For the southern Aegean islands in the late Second and Third Palace periods, an age of intensifying trans-Mediterranean linkage and expanding political units, there may often have been precariously little middle ground to hold between the two poles of (i) high profile connectivity, wealth and population, or (ii) an obscurity and relative poverty in terms of population and access to wealth that did not carry with it even the compensation of safety from external groups”.

We note that these rapid collapses are not induced by volatility but correspond to a smooth buckling of the landscape. This is reminiscent of the (often misapplied) catastrophe theory of the 70’s. The introduction of volatility would make the situation even more complicated which is a topic for future analytic investigations.

V. NUMERICAL SIMULATIONS

We are beginning to apply the models discussed above to realistic data using Monte Carlo methods to find suitable networks. However, \textit{a priori} it is difficult to make sensible estimates for the model parameters so we have to search for robust ranges where features are visible, much as we have to choose the right scale and coverage
when choosing a map for a problem in real life. We take a collection of 34 sites significant in the MBA including representatives from Crete, the Cyclades, the Dodecanese, Asia Minor and mainland Greece, as shown in Figure 4.

One objection to this is that the archaeological data may well be patchy; we may miss sites or not know their true size in this era. By allowing sites to choose find their optimal size $v_i$, we can avoid some of these problems. Later extensions could allow for the inclusion of speculative physical locations unsupported by data (so called `Atlantis sites') in order to test the likelihood of occupation. Alternatively a systematic approach to site location could be used such as the cultivatable land/population density method used by Broodbank.

The majority of work with archaeological networks has simple networks with no values or directions assigned to edges and with vertices carrying just their geographical location of sites. The PPA (Proximal Point Analysis) is typical (see Broodbank 2000 for examples and references). The analysis is often based on local properties of the vertices such as the degree and for rowing based societies it has been argued that this is all that is appropriate. Our networks are more complex, with sites and edges carrying additional values in order to capture the more hierarchical nature of the MBA Aegean and justified by larger data sets available. Figures 5 and 6 compare PPA and a similar density of edges in a Monte Carlo run and clearly show how links such as the Cyclades-Crete link which are around 100km long and on the edge of being feasible are missed by PPA but can play a vital role in a Monte Carlo simulation.

In terms of analysis the complexity of our networks provides several challenges. For instance the degree of a vertex is no longer a useful measure as edges are likely to carry a non-zero weight. For visualisation we can use a cut-off, and in our figures we do not show edges or vertices which are below 10% the size of the largest in that network. We could use a similar threshold method to map our network onto a simple graph upon which we could use the measures exploited in the existing archaeological literature. However the raison d'être of our work is precisely to exploit this as a feature. Thus we have to introduce new methods to the networks of island archaeology.

We will focus in this paper on ideas based around diffusion. Imagine a random walker who moves from site to site. At each time step the walker must follow an edge, respecting their directionality, choosing which edge to follow in proportion to the weight of the edge, $(S_i v_j e_{ij})$ if moving from site i to site j. This is a Markov process where the probability of being at a site i at time t is given by $r_i(t)$ and the vector $r(t)$ evolves as $r(t) = (\Pi)^t r(0)$. Here $\Pi$ is the transition matrix where

\[ \Pi_{ij} = S_i v_i e_{ji}. \]

This is basic idea behind PageRank used by Google to rank web pages (Brin and Page 1998). In practice a walker can get stuck in a dead end so we need to adapt this approach. Our present method is that if a dead end is reached, the walker starts again from a random vertex chosen with probability proportional to the weight $(S_i v_j)$ of the vertex. An example of the result is shown in Figure 7. This clearly identifies which sites are truly peripheral, such as Paroikia in the Cyclades which is not close to the route between Crete and Dodecanese. It also shows a hierarchy of Crete, the Dodecanese and then the Cyclades. Note that this also illustrates how one can emphasise the relational aspects over the physical locations as we use a non-geographical layout in Figure 7 (the Kamada-Kawai scheme as implemented in pajek, see references for details).
We can use diffusion in more ways. Suppose we want to understand which sites are dominant, be it in a cultural or political sense. There are examples of this in the archaeological literature. For instance the Renfrew tent model (see Cherry 1987 for a discussion) uses a simple geometric picture based on physical separation to determine zones of influence. While physical distance is an important factor, we have already built this into the assignment of weights to links so let us again exploit a random walker to access the global shape of the network. If we start a random walker from site i then we could ask how often it visited each site in the network. After a long time this would tend give the same answer as the ranking algorithm so at each step we restart the walk from site i with probability \( p \). The average walk length is then \( \frac{1-p}{p} \). The frequencies of visits to a site \( j \) when scaled by the weight of starting site \( i \), \( \mathbf{S}_i \mathbf{v}_i \), gives us a measure of the influence of site \( i \) on any other site \( j \). This is an influence matrix which can be used as the basis of block modelling (for example see de Nooy et al. 2005 for details). Here let us just associate each site to the site that has the largest influence over it. Note that the method does assign the influence of a site on itself. An example is shown in Figure 8 which indicates that for short ranges, 1.0, only Eastern and Western Crete form large regions dominated by Knossos and Gournia. Note that Malia remains independent suggesting that the link to the Cyclades via Thera is crucial. It is no coincidence that the next biggest group is in the Cyclades including Thera. As we increase the range nothing happens until at about 1.5 there is a dramatic shift to three groups: Ios dominates the Cyclades, Miletus dominates the Dodecanese, and Gournia dominates Crete and all other outlying sites including Rhodes. A number of interpretations are possible; one is that this represents zones of weaker influence or alternatively this would be the pattern if sites had the power to exert their influence further.

VI. CONCLUSIONS

To date our work merely indicates the possibilities of these techniques. Social Science has had much experience analysing such networks and there are now many new tools available post-Watts and Strogatz. The results of our models have yet to be correlated with the data, such as ceramic data. Again Broodbank sets an example for the EBA. In particular we will have to make sure any conclusions are robust against changes in the details of our models.

There are numerous areas where improvements are already being made. We have to adapt our input distances \( d_{ij} \) to reflect actual transport times rather than physical distances. The list and size of sites can be fitted to archaeological data, or we could adapt Broodbank’s method of assigning sites on the basis of cultivatable area. Within the model we have variations where we use network distances rather than pure physical distances \( d_{ij} \), both within the Hamiltonian and in the analysis. Finally we expect to move from static configurations to study problems of time evolution. This could be slow ‘adiabatic’ changes, such as population build up or quick ‘quenches’. In either case, it is quite possible that the system gets stuck for a time in a meta-stable state with the instability only apparent much later. This might be a good model for the transition from Middle to Late Bronze Age, from the Minoan to Mycenaean eras. Figure 9 shows how one might compare the MBA Aegean with and without Thera.

REFERENCES


PAJEK, Program for Large Network Analysis, http://vlado.fmf.uni-lj.si/pub/networks/pajek/
FIGURES

Figure 1: The energy landscape for small \( \lambda / \kappa \) with the vertical axis \( H \) and horizontal axes \( v \) and \( e \). In this regime sites appear to be close to their optimal size and edges can have non-trivial values.

Figure 2: The energy for large \( \lambda / \kappa \) with the vertical axis \( H \) and horizontal axes \( v \) and \( e \). Now the network is forced to extremal values.

Figure 3: The energy landscape for fixed fixed \( \kappa \) and \( \mu \). The mean field average of \( v \) (vertical axis) is shown against varying \( \lambda \) and \( j \).
Figure 4: The Aegean with the location of the 34 MBA sites used. Crete is at the bottom, the Cyclades are the islands in the centre around Ios (25). They also contain Akrotiri (10) on Thera which exploded c. 1600 BC. The Dodecanese run from Rhodes (16) to Kalymnos (31), off the coast of Asia Minor. Mycenae (27) becomes dominant in the Late Bronze Age, c 1400 – 1200 BC.
Figure 5: PPA analysis with three outgoing edges to nearest three sites. Colour indicates sites of equal degree (Miletus and Myndus have largest degree of 6, white; pink =5; blue=4 incl. Knossos; red=3; green=2 incl. Malia; yellow=1 incl. Kastri). The size of the vertex is proportional to betweenness, a measure of the number of shortest paths passing through each vertex. Knossos, Malia and Kastri are the most central.

Figure 6: Monte Carlo analysis for $\kappa=2.0$, $\lambda=1.0$, $\mu=0.35$ and $\eta=0.7$. Vertices are coloured by strength, the total weight of the in and out going edges: largest are Gournia, Malia and Knossos followed closely by the rest of central Crete. The Dodecanese are about half the strength and the Cyclades are a third the strength. The vertex size is given by the betweenness and this shows a very different story with sites on the edges of clusters scoring highly. This includes Malia and Knossos but now the Cyclades scores even higher than these indicating their central role.
Figure 7: A nongeographical display for same values as Figure 6. Sites ranked using diffusion model, size of vertex proportional to ranking. Central Cretan sites are ranked most highly. Sites labelled by their numbers given in Figure 4.

Figure 8: The pattern of dominance for p=0.5 for the same network as in Figure 6. Note that Crete splits into a Western and Eastern region dominated by Knossos and Gournia respectively. Only Malia is strong enough to remain independent. The pattern only changes at p=0.6 when suddenly three groups emerge: Ios dominates the Cyclades, Miletus dominates the Dodecanese, and Gournia dominates Crete and all other outlying sites including Rhodes. The vertex size is proportional to the vertex weight, the largest sites are three times as big as the smallest.
Figure 9: The same values are used but now on the left Thera is removed while it remains in place on the right. The weights of sites is similar but now the Dodecanese is ranked far higher. Vertices coloured by their weights.