A model of cooperation for bipartite networks

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http://sbs-xnet.sbs.ox.ac.uk/complexity/
Growth and Agglomeration

The internet, 29 June 1999
The Internet Mapping Project
http://www.cheswick.com/ches/map/index.html
The New York City Garment Industry
Data collected by UNITE

- Extensive data set of ~700,000 customer-supplier transactions in the NY garment industry from 1985-2003 collected by UNITE.

<table>
<thead>
<tr>
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<th>periodto</th>
<th>adjgr</th>
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</table>
The NYGI Network Structure
Contracting behaviour

1985: 3249 firms

1995: 1046 firms

2003: 190 firms
The Microscopic Dynamics

- Order of magnitude contraction in firm population
- High rates of exit and entry
- High rate of breaking and forming ties
Stationary Degree Distribution
Evolution of Disassortativity
Error rates

Percentage of links with refund transactions for each year.
Robustness
A model for network decline

i:  
\[
\begin{array}{c}
\text{1} \quad \text{3} \\
\text{2} \quad \text{4} \quad \text{5} \\
\text{6}
\end{array}
\rightarrow
\begin{array}{c}
\text{1} \quad \text{3} \\
\text{2} \quad \text{4} \quad \text{5} \\
\text{6}
\end{array}
\]

Node deletion

ii:  
\[
\begin{array}{c}
\text{2} \\
\text{4} \\
\text{5} \\
\text{6}
\end{array}
\rightarrow
\begin{array}{c}
\text{2} \\
\text{4} \\
\text{5} \\
\text{6}
\end{array}
\]

a: Removing \( p_{ij} \)  
\[
\begin{array}{c}
\text{2} \\
\text{4} \\
\text{5} \\
\text{6}
\end{array}
\rightarrow
\begin{array}{c}
\text{2} \\
\text{4} \\
\text{5} \\
\text{6}
\end{array}
\]

b: Replacing \( q \)

iii:  
\[
\begin{array}{c}
\text{2} \\
\text{4} \\
\text{5} \\
\text{6}
\end{array}
\rightarrow
\begin{array}{c}
\text{2} \\
\text{4} \\
\text{5} \\
\text{6}
\end{array}
\]

Growth mechanism: PA
Declining trajectories over the years for manufacturers (blue circles), contractors (red diamonds), and hybrids (green squares). The number of firms is normalized to its corresponding value in 1985. The inset shows the relationship in a log-log scale between the total number of firms and links in the network. The solid line is the fit to the data defined by $\gamma=1.22\pm0.01$ (s.e), $r^2=0.99$. 

Population dynamics
Trophic web of species from the El Verde Ranforest, Carribean National Fores, Puerto Rico.
http://www.foodwebs.org
Simple rules for food webs

Predators consume contiguous sequence of prey in a one-dimensional trophic niche. (Williams and Martinez, Nature 2000)

- S, L \[\rightarrow\] input parameters
- A (S×S) \[a_{ij}=1\] if species \(j\) consumes species \(i\)

1. Species are assigned niche values that form a totally ordered set.
2. Species have an exponentially decaying probability of preying on species with lower niche values.
Mutualistic networks

<table>
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<th></th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
<th>P₄</th>
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<tr>
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<td>A₅</td>
<td>1</td>
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</table>
Structuring mutualistic interactions

- Trait complementarity
- Exploitation barriers
- Hierarchical phylogenetic relationships

Reward traits $t_{R_p}$

Foraging traits $t_{R_a}$

Modulated by Environmental Pressure

Geographic variation
Population diversity

Temporal variation
Population density
Bipartite cooperation model - specialization

A, P, L \(\Rightarrow\) input parameters

1. **Specialisation Rule**

Fix number of partners \(l_p\) from \(a \in A\) that each \(p \in P\) interacts with

\[
l_p = 1 + \text{Round}((L - P) \frac{t_{Rp} \lambda_p}{\sum_j t_{Rj} \lambda_j})
\]

\(t_{Rp}\) – draw from uniform distribution \([0,1]\) analogous to niche value

\(\lambda_p\) – draw from exponential distribution \(p(x)=\beta e^{-\beta x}\) where

\[
\beta = \frac{P(A-1)}{2(L-P)} - 1
\]
Bipartite cooperation model - interaction

2. Interaction Rule \((t_{Rp} \leftrightarrow t_{Fa})\)

Which members from \(a \in A\) cooperate with each member of \(p \in P\)?

Sort \(P\) in ascending order by reward traits
Sort \(A\) in descending order by foraging traits

Link nodes in \(P\) sequentially to unlinked nodes in \(A\) subject to \(t_{Rpi} > \lambda_{lpi}\)
If \(t_{Rpi} \leq \lambda_{lpi}\) link instead to randomly chosen node in \(A\) with previous links

\(t_{Fa}\) – draw from uniform distribution \([0,1]\)

\(\lambda_{lp}\) – draw from exponential distribution
## Model vs. empirical data

<table>
<thead>
<tr>
<th>Dataset – Environment</th>
<th>$L$</th>
<th>$P$</th>
<th>$A$</th>
<th>($KS_P-KS_A$)</th>
<th>$N$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marsh, Japan</td>
<td>430</td>
<td>64</td>
<td>187</td>
<td>0.326++ -- 0.438++</td>
<td>0.976++ (0.969)</td>
<td>0.551++ (0.553)</td>
</tr>
<tr>
<td>Grassland, Cass, New Zealand</td>
<td>374</td>
<td>41</td>
<td>139</td>
<td>0.633++ -- 0.385++</td>
<td>0.957++ (0.960)</td>
<td>0.474+ (0.465)</td>
</tr>
<tr>
<td>Subalpine forest/meadow, Japan</td>
<td>865</td>
<td>90</td>
<td>354</td>
<td>0.552++ -- 0.001*</td>
<td>0.985+ (0.976)</td>
<td>0.545+ (0.532)</td>
</tr>
<tr>
<td>Subalpine, Arthur’s, New Zealand</td>
<td>120</td>
<td>18</td>
<td>60</td>
<td>0.108+ -- 0.999++</td>
<td>0.858* (0.936)</td>
<td>0.553** (0.527)</td>
</tr>
<tr>
<td>Subalpine, Craigieburn, New Zealand</td>
<td>346</td>
<td>49</td>
<td>118</td>
<td>0.002* -- 0.001*</td>
<td>0.961++ (0.955)</td>
<td>0.480+ (0.468)</td>
</tr>
<tr>
<td>Tundra, Canada</td>
<td>179</td>
<td>29</td>
<td>81</td>
<td>0.097+ -- 0.989++</td>
<td>0.971+ (0.950)</td>
<td>nm</td>
</tr>
<tr>
<td>Scrub/snow gum forest, Australia</td>
<td>252</td>
<td>36</td>
<td>81</td>
<td>0.608+ -- 0.076+</td>
<td>0.935++ (0.949)</td>
<td>nm</td>
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<tr>
<td>Deciduous forest, USA</td>
<td>65</td>
<td>7</td>
<td>33</td>
<td>0.911++ -- 0.642+</td>
<td>0.953+ (0.930)</td>
<td>nm</td>
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<tr>
<td>Arctic tundra, Greenland</td>
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<td>31</td>
<td>75</td>
<td>0.038** -- 0.118+</td>
<td>0.793* (0.914)</td>
<td>nm</td>
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<tr>
<td>Subarctic rock slope, Sweden</td>
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<td>24</td>
<td>118</td>
<td>0.223+ -- 0.005**</td>
<td>0.927+ (0.952)</td>
<td>nm</td>
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<tr>
<td>NYGI 1985</td>
<td>7250</td>
<td>823</td>
<td>2562</td>
<td>0.061+ -- 0.115+</td>
<td>0.997+ (0.996)</td>
<td>0.598* (0.502)</td>
</tr>
<tr>
<td>NYGI 1991</td>
<td>3981</td>
<td>325</td>
<td>1590</td>
<td>0.101+ -- 0.531++</td>
<td>0.994++ (0.993)</td>
<td>0.601* (0.529)</td>
</tr>
<tr>
<td>NYGI 1997</td>
<td>1450</td>
<td>148</td>
<td>700</td>
<td>0.003** -- 0.264+</td>
<td>0.990+ (0.988)</td>
<td>0.653** (0.625)</td>
</tr>
<tr>
<td>NYGI 2003</td>
<td>228</td>
<td>62</td>
<td>128</td>
<td>0.370+ -- 0.002**</td>
<td>0.976+ (0.969)</td>
<td>0.711+ (0.700)</td>
</tr>
</tbody>
</table>

For each pollination dataset and four organizational networks used in this paper, the table presents its environment/location; total number of links $L$, $P$ and $A$ are the number of nodes in class $P$ and class $A$ respectively. Note that all networks have a ratio $P/A<0.5$, which has been found to be an important factor limiting the appearance of scale-free distributions\(^2\). For the degree distributions, ($KS_P-KS_A$) shows the combined Kolmogorov-Smirnov (KS) probability using the two-group equivalence KS test between the empirical and model-generated distributions for class $P$ and class $A$ respectively. $N$ and $Q$ correspond to the observed nestedness and mean modularity values respectively, along with the normalized errors ($z$-scores) for the comparison between the empirical and model-generated values. The model-generated mean values for $N$ and $Q$ are shown inside the parentheses. Five of the observed pollination networks have already been found to be non-modular (nm)\(^10\). All comparisons are based on 1000 model simulations.
Cumulative degree distributions

• plants / manufacturers

+ animals / contractors
Rescaled degree distributions

Figure a shows the cumulative degree distribution $P_{cum}(k)$ for members of class P (plants, manufacturers), and Figure b $P_{cum}(k)$ for members of class A (animals, contractors). The number of partners $k$ is scaled by a multiplicative factor of $1/z_P$ for members of P, and $1/z_A$ for members of A, where $z_P = L/P$ and $z_A = L/A$. Solid symbols correspond to pollination networks and open symbols to organizational networks. Note that all distributions collapse into a single curve. The solid line corresponds to the model-generated distributions averaged over 1000 simulations.
Nestedness

Nestedness. **a** and **b** show for the NYGI (1985-2003) and the pollination networks respectively, the observed nestedness values (dashed line), and the average plus two standard deviations values generated by the model (yellow dots) and the random assemblages (green crosses) following Bascompte’s et al.\textsuperscript{25} null model. Here, a nestedness of 0 means a perfectly nested matrix. Note that the two standard deviation bars account for \(-2<Z\)-score<2 as defined by \(Z=\text{(observed-average predicted)/st. dev. predicted.}\) This analysis was carried out over 1000 simulations.
Connectivity roles. a and b show for the NYGI (1985-2003) and the modular pollination networks respectively, the percentage of population for the observed connectivity roles (colors), and the values (avg±stdv) generated by the model (black bars). We followed Guimera’s et.al. classification for nodes, where roles 1-4 are classified as non-hubs and roles 5-7 as hubs.
Conclusions

- Simple stochastic model which generates many of the structural features of mutualistic networks
- Unexpected correspondence between:
  \( \text{Ecological Networks} \leftrightarrow \text{Organisational networks} \)
- Other examples where bipartite cooperation model applies?
- Need more systematic approach for assessing goodness of fit.

References:
Serguei Saavedra, Felix Reed-Tsochas and Brian Uzzi (2008), "Asymmetric disassembly and robustness in declining networks", *Proceedings of the National Academy of Sciences*, in press.
From fashion to Broadway?
Thank you