

<http://sbs-xnet.sbs.ox.ac.uk/complexity/>

# A model of cooperation for bipartite networks



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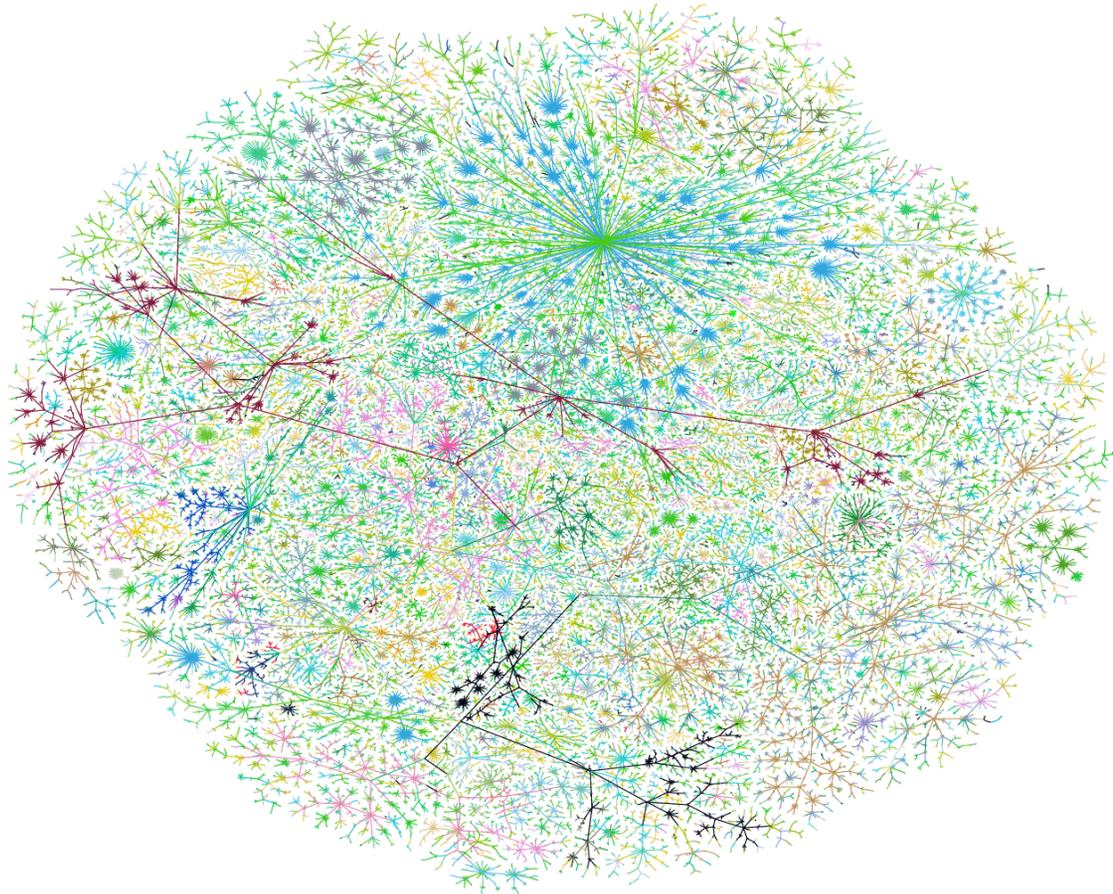
Kellogg School of Management and Northwestern Institute on Complex Systems,  
Northwestern University



CABDyN Seminar

14 October 2008

# Growth and Agglomeration

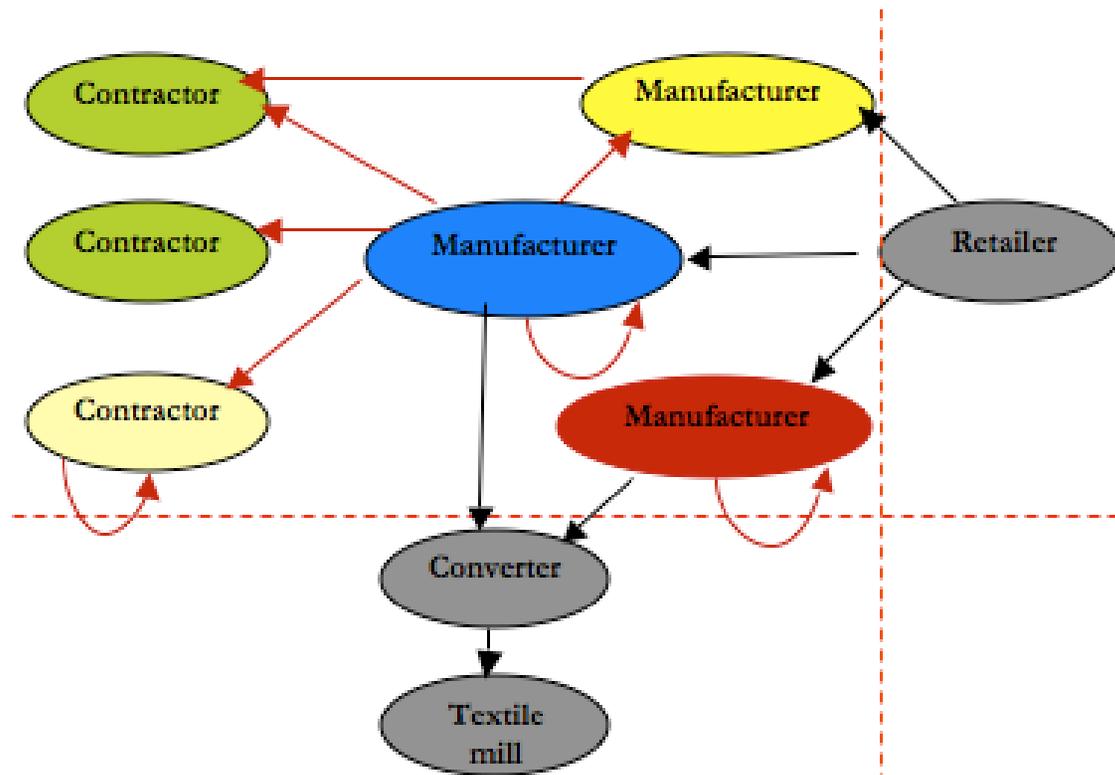


The internet, 29 June 1999  
The Internet Mapping Project  
<http://www.cheswick.com/ches/map/index.html>



# The New York City Garment Industry

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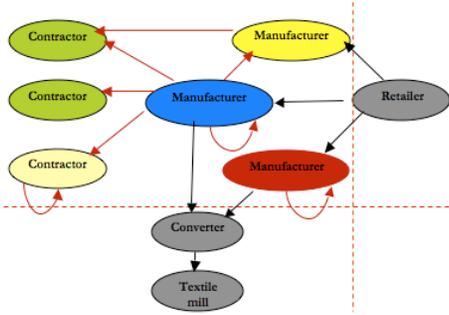


# Data collected by UNITE

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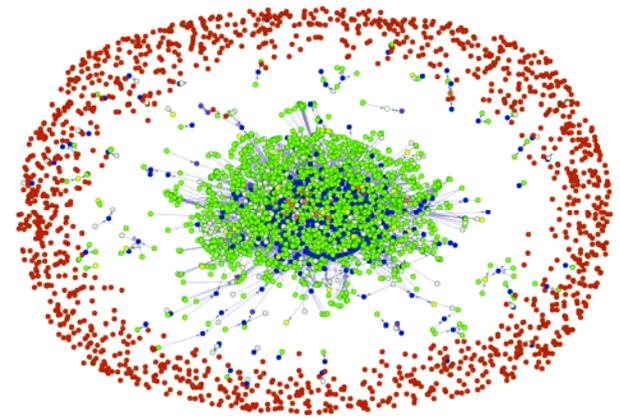
- Extensive data set of ~700,000 customer-supplier transactions in the NY garment industry from 1985-2003 collected by UNITE.

	paidbyfi	paidforf	periodfr	periodto	adjgr	gross	net
1	00109332	00109332	010185	013185	2881.23	2881.33	605.8
2	00109332	00109332	100185	103085	189.63	189.65	41.77
3	00109332	01000019	010185	013185	706.66	706.68	148.58
4	00109332	00106409	010185	053185	6049.93	6049.96	953.55
5	00109332	00107128	090185	093085	4356.48	4356.5	719.53
6	00109332	00205734	040185	053185	-1440.02	-1439.99	-226.96
7	00109332	00109332	010185	063085	1338.38	1338.36	281.39



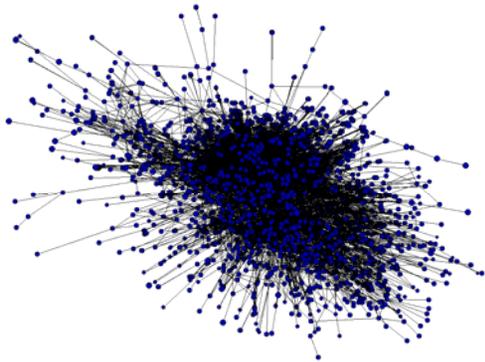
# The NYGI Network Structure

	paidbyfi	paidforf	periodfr	periodto	adjgr	gross	net
1	00109332	00109332	010185	013185	2881.23	2881.33	605.8
2	00109332	00109332	100185	103085	189.63	189.65	41.77
3	00109332	01000019	010185	013185	706.66	706.68	148.58
4	00109332	00106409	010185	053185	6049.93	6049.96	953.55
5	00109332	00107128	090185	093085	4356.48	4356.5	719.53
6	00109332	00205734	040185	053185	-1440.02	-1439.99	-226.96
7	00109332	00109332	010185	063085	1338.38	1338.36	281.39



# Contracting behaviour

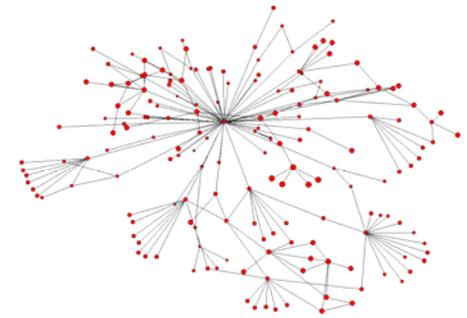
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**1985: 3249 firms**



**1995: 1046 firms**



**2003: 190 firms**

# The Microscopic Dynamics

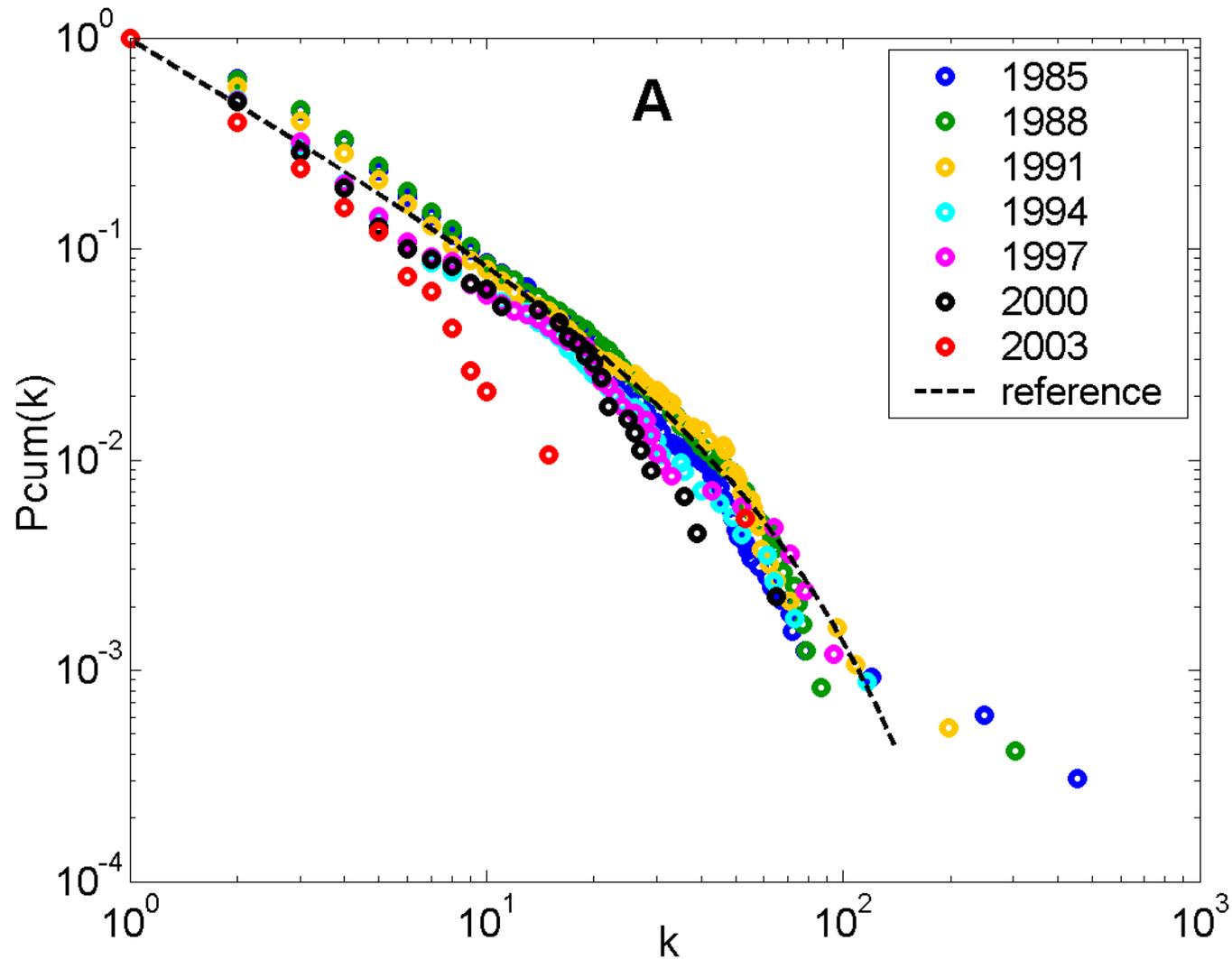
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- Order of magnitude contraction in firm population
- High rates of exit and entry
- High rate of breaking and forming ties

	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
<b>Firms</b>	3249	2832	2586	2410	2261	2086	1880	1745	1554	1135	1135	974	842	751	566	449	337	256	190
<b>DF</b>	X	799	737	686	650	615	609	543	588	583	680	348	349	268	318	237	182	147	101
<b>NF</b>	X	495	572	588	568	510	466	473	425	219	455	296	217	177	133	120	70	66	35
<b>Links</b>	7250	6070	5632	5504	4982	4596	3981	3498	3118	1917	2035	1775	1450	1236	915	716	458	334	228
<b>DL1</b>	X	1924	1613	1507	1362	1144	1171	1091	1108	1241	1259	591	567	407	505	363	291	185	122
<b>NL1</b>	X	938	1177	1161	1092	996	838	854	748	377	450	507	322	83	221	190	86	79	100
<b>DL2</b>	X	1790	1585	1410	1563	1474	1281	1020	756	698	736	282	307	257	173	127	102	63	42
<b>NL2</b>	X	1596	1583	1628	1311	1236	999	774	736	361	372	309	227	173	136	101	49	45	18

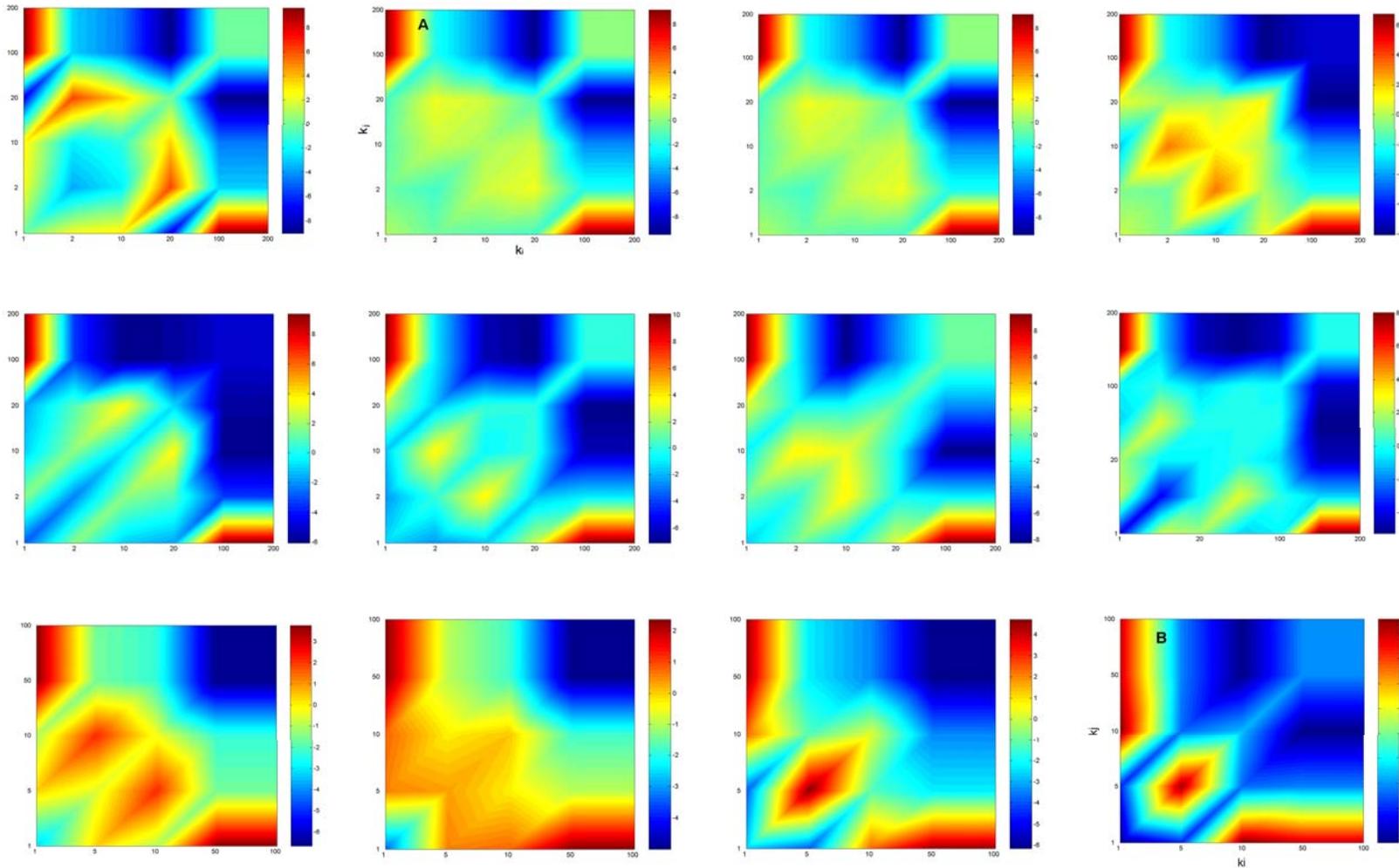
# Stationary Degree Distribution

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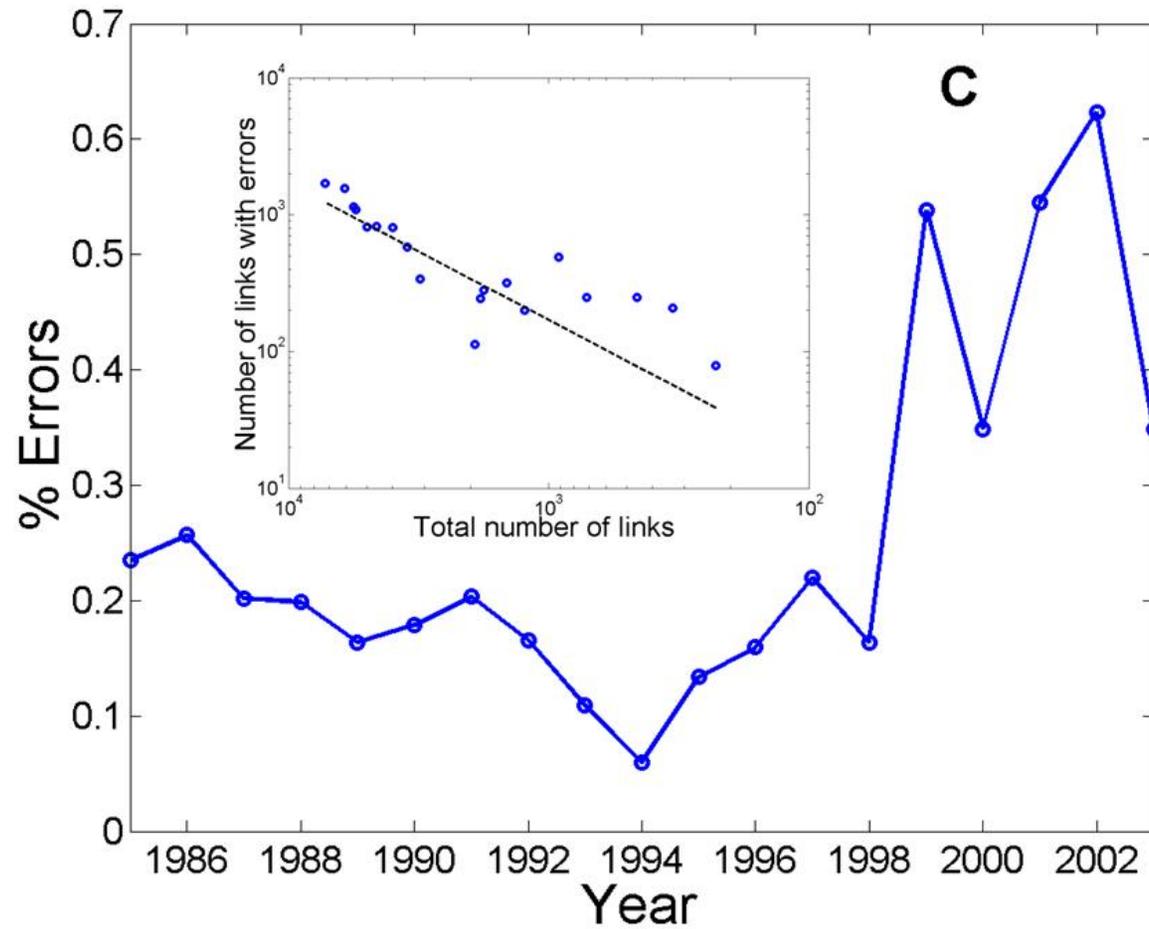


# Evolution of Disassortativity

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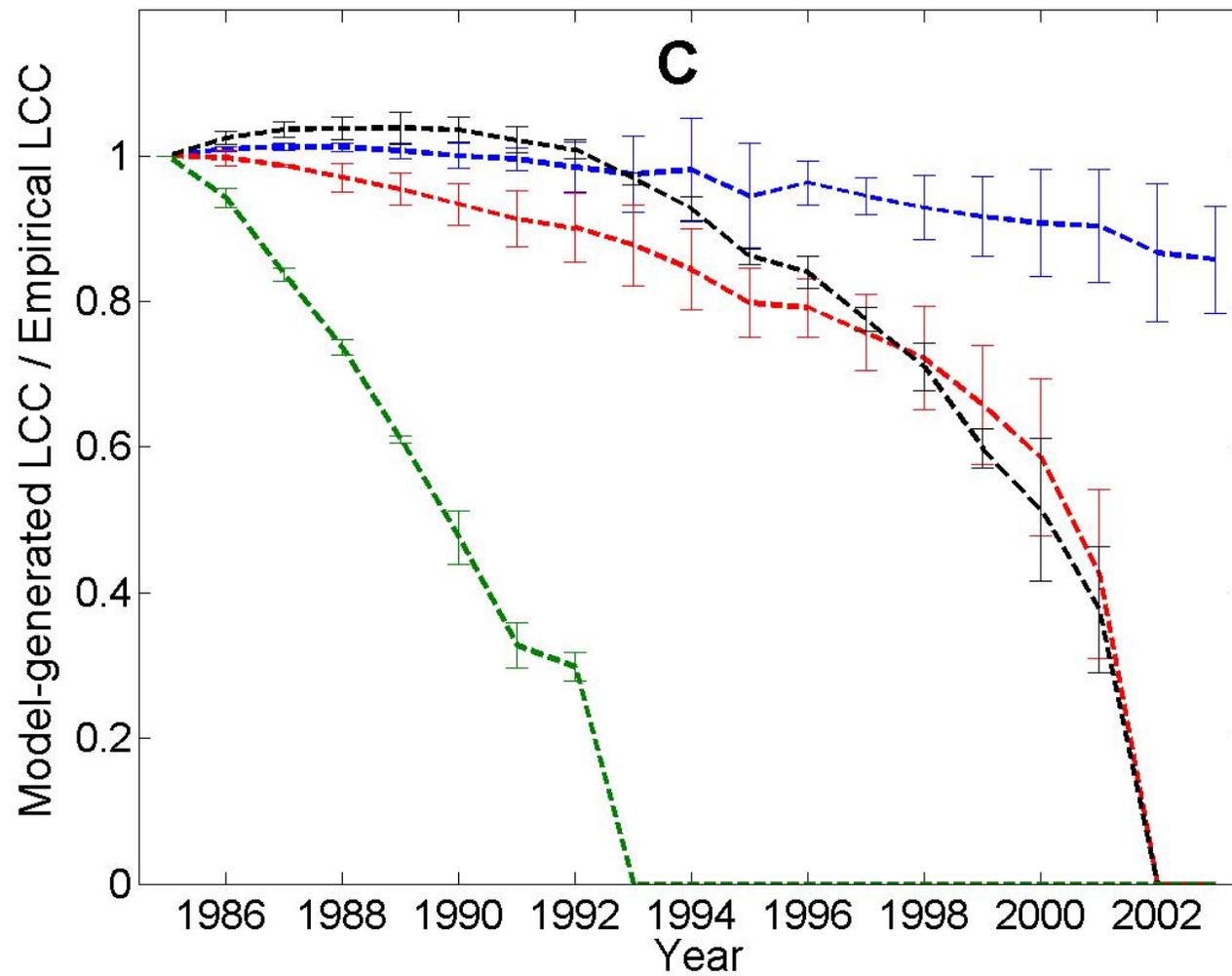


# Error rates

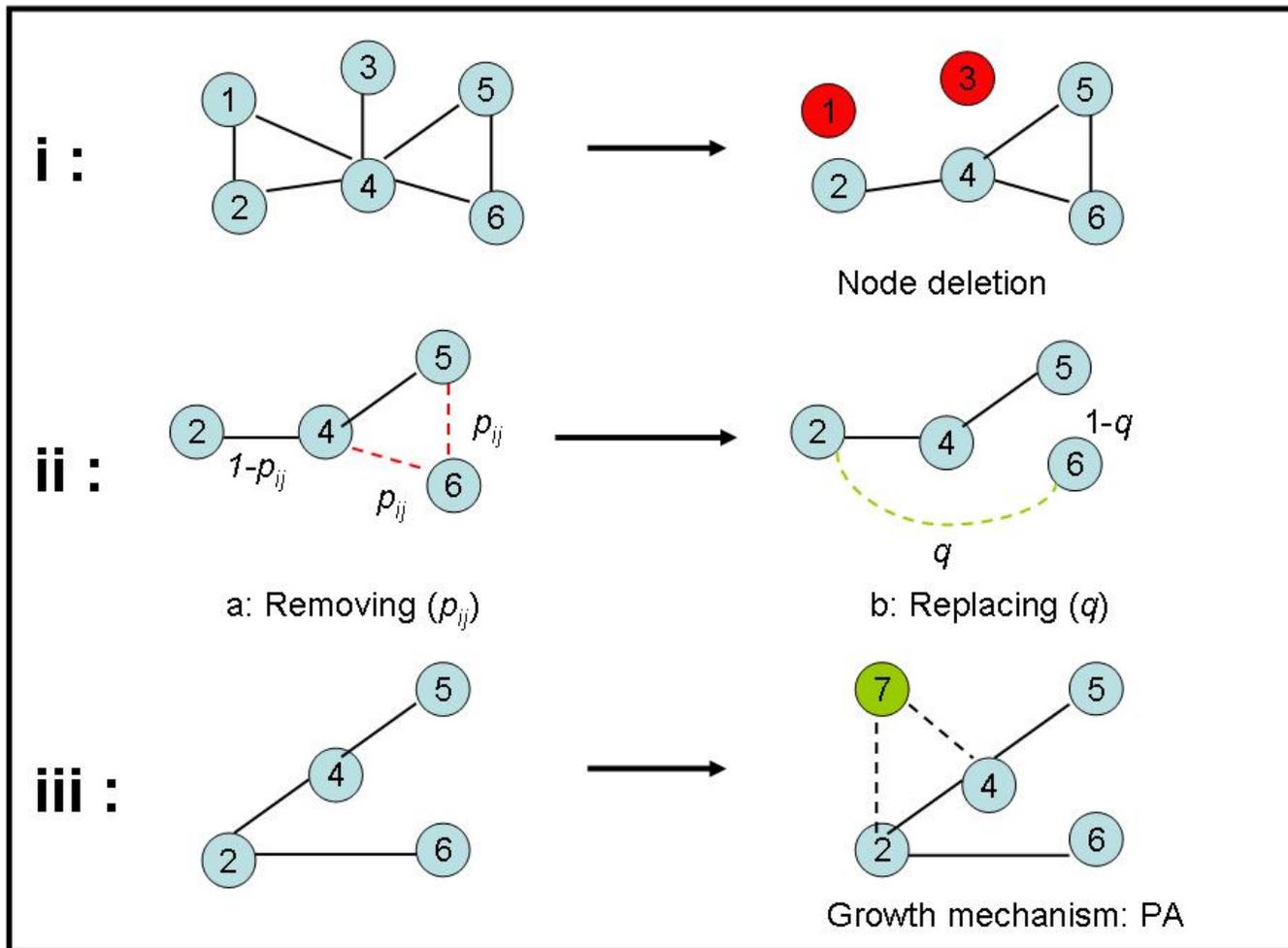


Percentage of links with refund transactions for each year.

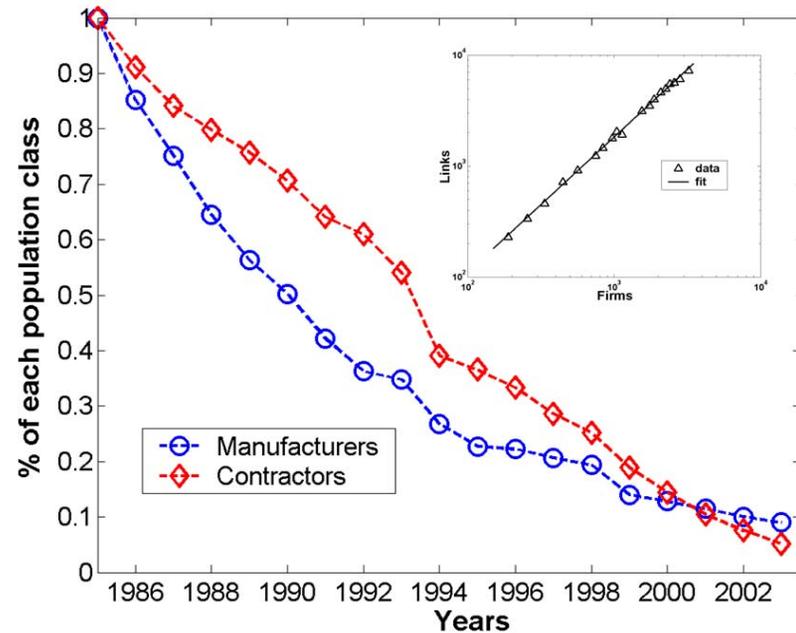
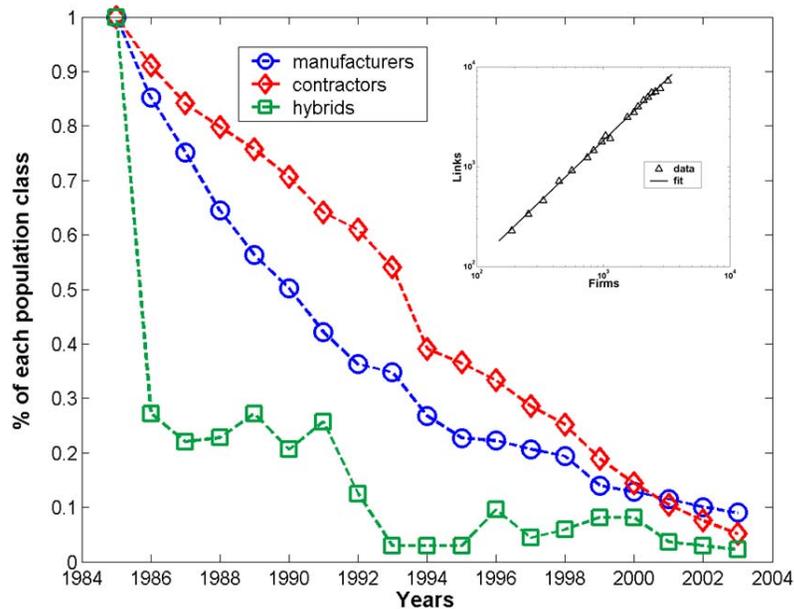
# Robustness



# A model for network decline



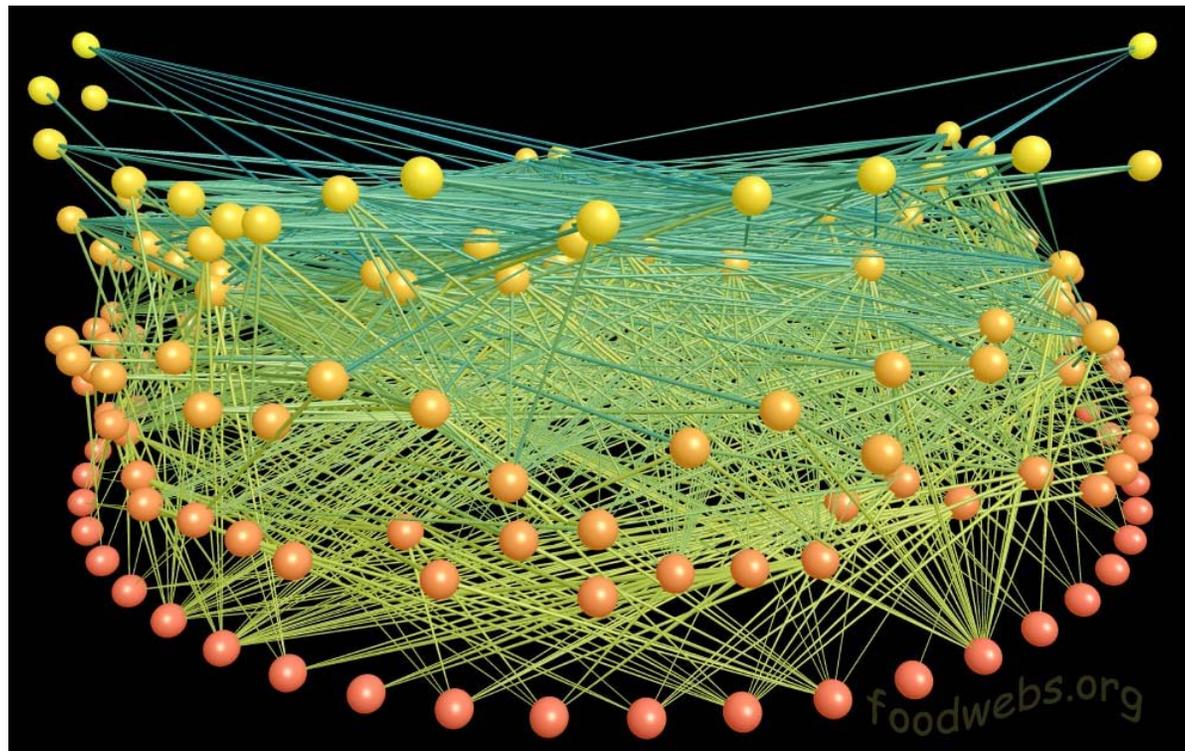
# Population dynamics



Declining trajectories over the years for manufacturers (blue circles), contractors (red diamonds), and hybrids (green squares). The number of firms is normalized to its corresponding value in 1985. The inset shows the relationship in a log-log scale between the total number of firms and links in the network. The solid line is the fit to the data defined by  $\gamma=1.22\pm 0.01(\text{s.e.})$ ,  $r^2=0.99$ .

# Food webs

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Trophic web of species from the El Verde Ranforest,  
Carribbean National Fores, Puerto Rico.  
<http://www.foodwebs.org>

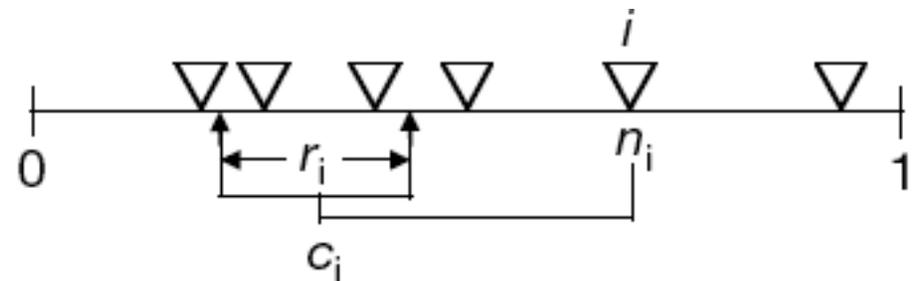
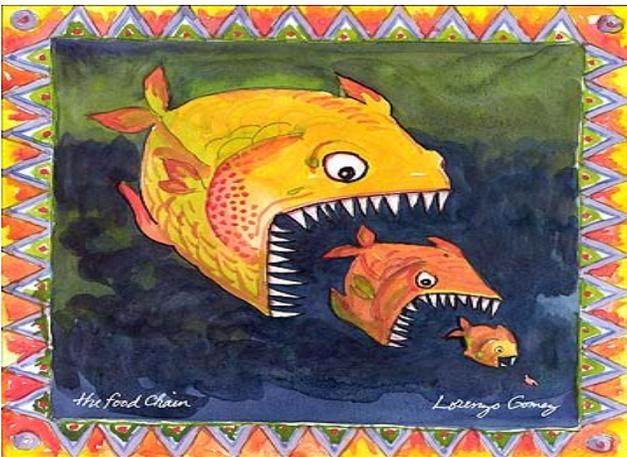
# Simple rules for food webs

Predators consume contiguous sequence of prey in a one-dimensional trophic niche. (Williams and Martinez, *Nature* 2000)

$S, L \rightarrow$  input parameters

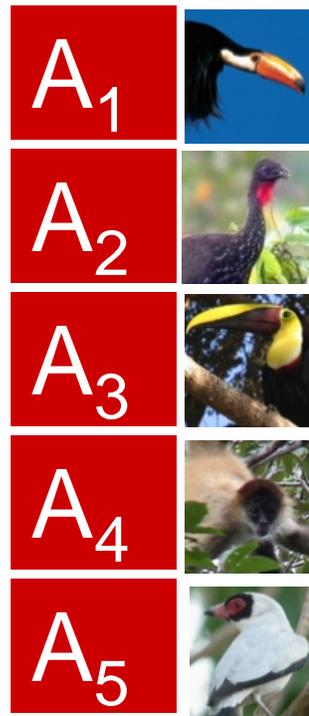
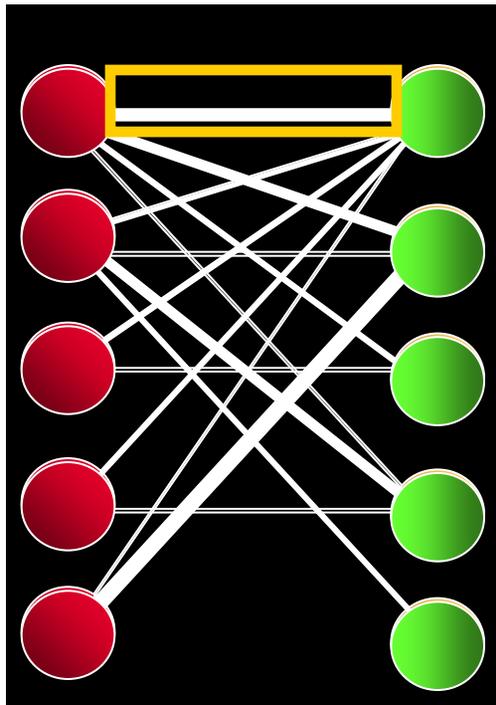
$A (S \times S) \quad a_{ij}=1$  if species  $j$  consumes species  $i$

- (1) species are assigned niche values that form a totally ordered set
- (2) species have an exponentially decaying probability of preying on species with lower niche values



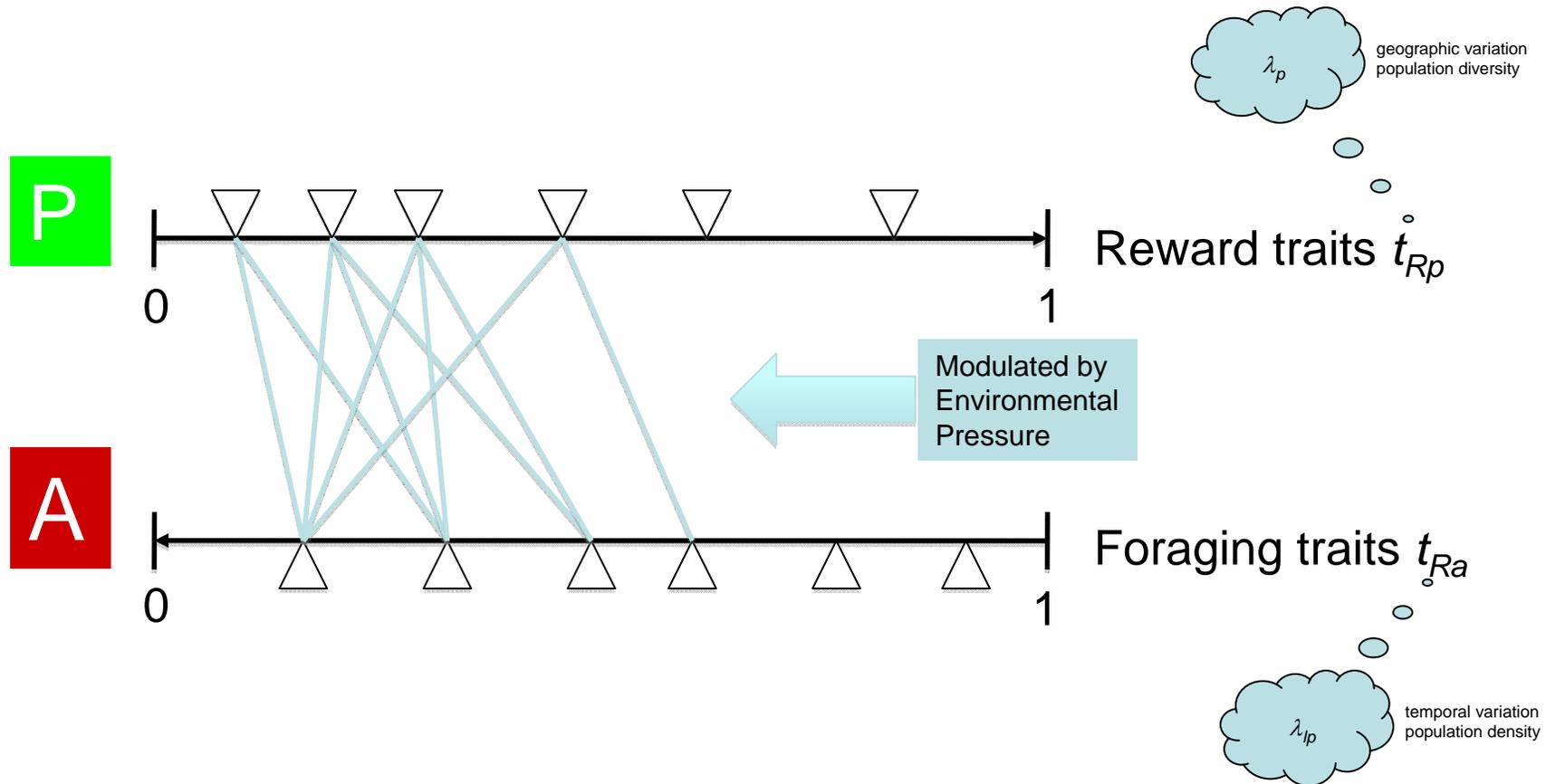


# Mutualistic networks



$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
				
1	1	1	1	1
1	1	1	0	0
0	1	0	0	0
1	0	0	0	0
1	0	0	0	0

# Structuring mutualistic interactions



- Trait complementarity
- Exploitation barriers
- Hierarchical phylogenetic relationships

# Bipartite cooperation model - specialization

A, P, L → input parameters

## 1. Specialisation Rule



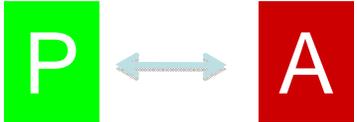
Fix number of partners  $l_p$  from  $a \in A$  that each  $p \in P$  interacts with

$$l_p = 1 + \text{Round}\left((L - P) \frac{t_{Rp} \lambda_p}{\sum_j t_{Rj} \lambda_j}\right)$$

$t_{Rp}$  – draw from uniform distribution  $[0,1]$  *analogous to niche value*

$\lambda_p$  – draw from exponential distribution  $p(x) = \beta e^{-\beta x}$  where  $\beta = \frac{P(A-1)}{2(L-P)} - 1$

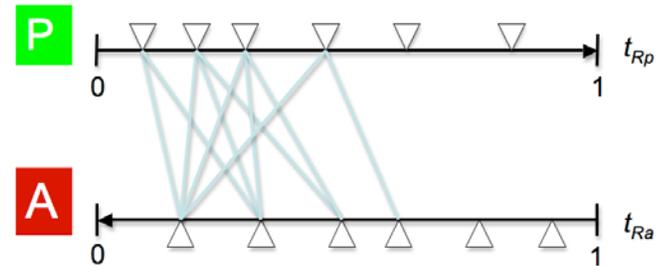
# Bipartite cooperation model - interaction

2. Interaction Rule ( $t_{Rp} \leftrightarrow t_{Fa}$ ) 

Which members from  $a \in A$  cooperate with each member of  $p \in P$ ?

Sort  $P$  in ascending order by reward traits

Sort  $A$  in descending order by foraging traits



Link nodes in  $P$  sequentially to unlinked nodes in  $A$  subject to  $t_{Rpi} > \lambda_{lpi}$

If  $t_{Rpi} \leq \lambda_{lpi}$  link instead to randomly chosen node in  $A$  with previous links

$t_{Fa}$  – draw from uniform distribution  $[0,1]$

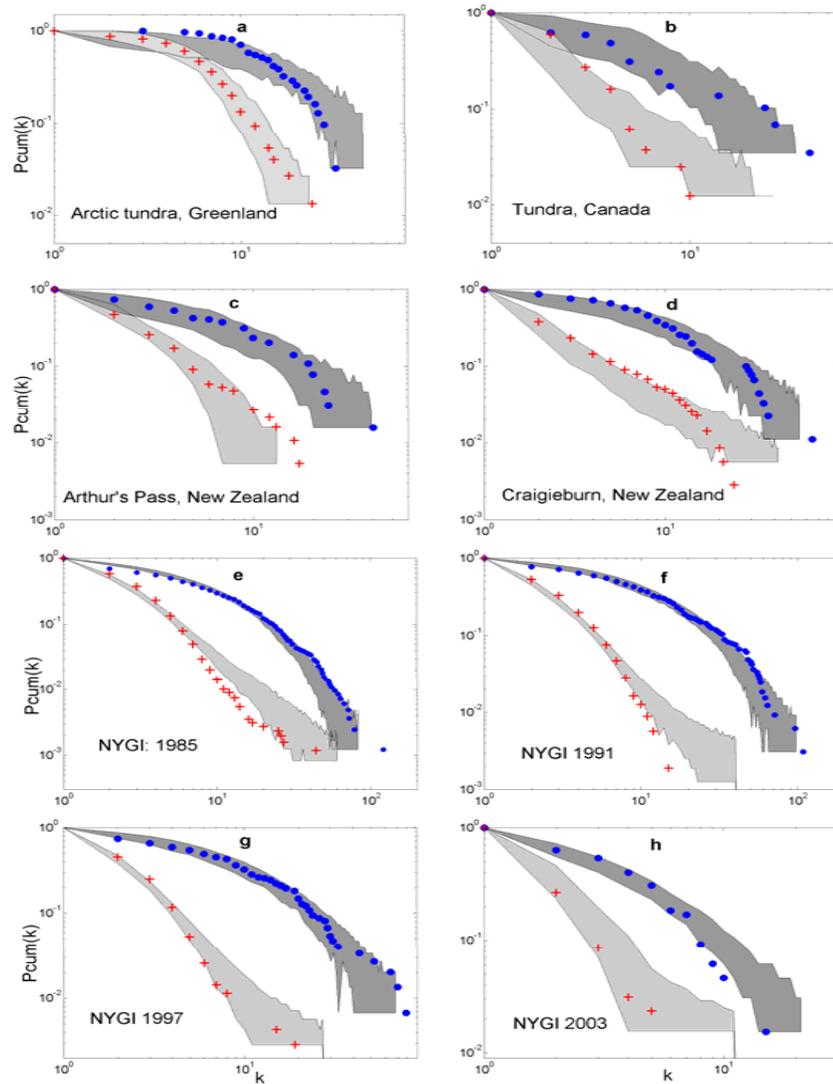
$\lambda_{lp}$  – draw from exponential distribution

# Model vs. empirical data

Dataset – Environment	$L$	$P$	$A$	$(KS_P - KS_A)$	$N$	$Q$
Marsh, Japan	430	64	187	0.326†† -- 0.438††	0.976†† (0.969)	0.551†† (0.553)
Grassland, Cass, New Zealand	374	41	139	0.633†† -- 0.385††	0.957†† (0.960)	0.474† (0.465)
Subalpine forest/meadow, Japan	865	90	354	0.552†† -- 0.001*	0.985† (0.976)	0.545† (0.532)
Subalpine, Arthur's, New Zealand	120	18	60	0.108† -- 0.999††	0.858* (0.936)	0.553** (0.527)
Subalpine, Craigieburn, New Zealand	346	49	118	0.002* -- 0.001*	0.961†† (0.955)	0.480† (0.468)
Tundra, Canada	179	29	81	0.097† -- 0.989††	0.971† (0.950)	nm
Scrub/snow gum forest, Australia	252	36	81	0.608†† -- 0.076†	0.935†† (0.949)	nm
Deciduous forest, USA	65	7	33	0.911†† -- 0.642††	0.953† (0.930)	nm
Arctic tundra, Greenland	453	31	75	0.038** -- 0.118†	0.793* (0.914)	nm
Subarctic rock slope, Sweden	242	24	118	0.223† -- 0.005**	0.927† (0.952)	nm
NYGI 1985	7250	823	2562	0.061† -- 0.115†	0.997† (0.996)	0.598* (0.502)
NYGI 1991	3981	325	1590	0.101† -- 0.531††	0.994†† (0.993)	0.601* (0.529)
NYGI 1997	1450	148	700	0.003** -- 0.264†	0.990† (0.988)	0.653** (0.625)
NYGI 2003	228	62	128	0.370† -- 0.002**	0.976† (0.969)	0.711† (0.700)

For each pollination dataset and four organizational networks used in this paper, the table presents its environment/location; total number of links  $L$ ,  $P$  and  $A$  are the number of nodes in class P and class A respectively. Note that all networks have a ratio  $P/A < 0.5$ , which has been found to be an important factor limiting the appearance of scale-free distributions<sup>22</sup>. For the degree distributions,  $(KS_P - KS_A)$  shows the combined Kolmogorov-Smirnov (KS) probability using the two-group equivalence KS test between the empirical and model-generated distributions for class P and class A respectively.  $N$  and  $Q$  correspond to the observed nestedness and mean modularity values respectively, along with the normalized errors (z-scores) for the comparison between the empirical and model-generated values. The model-generated mean values for  $N$  and  $Q$  are shown inside the parentheses. Five of the observed pollination networks have already been found to be non-modular (nm)<sup>10</sup>. All comparisons are based on 1000 model simulations.

# Cumulative degree distributions



• plants / manufacturers

+ animals / contractors

# Rescaled degree distributions

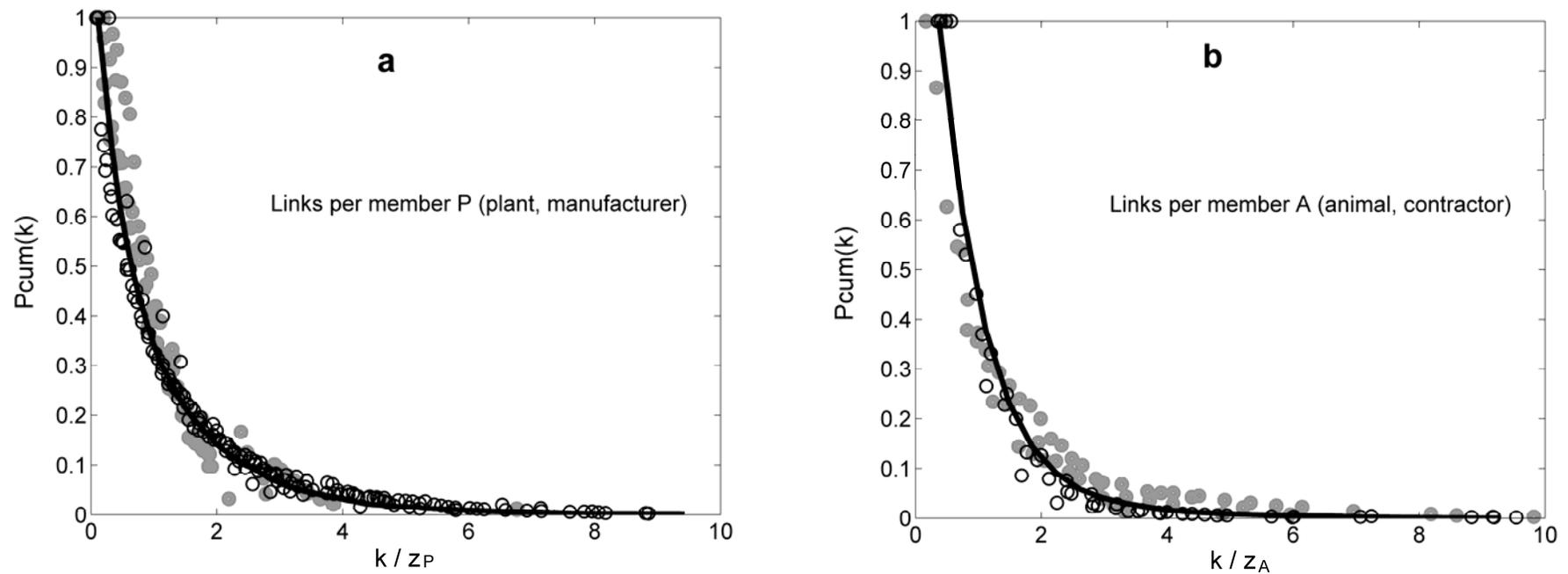
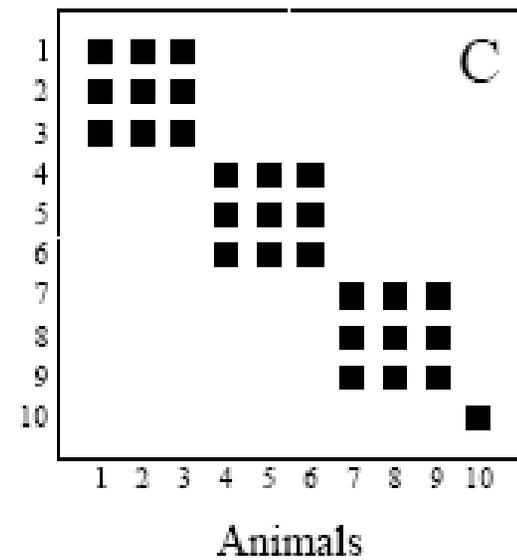
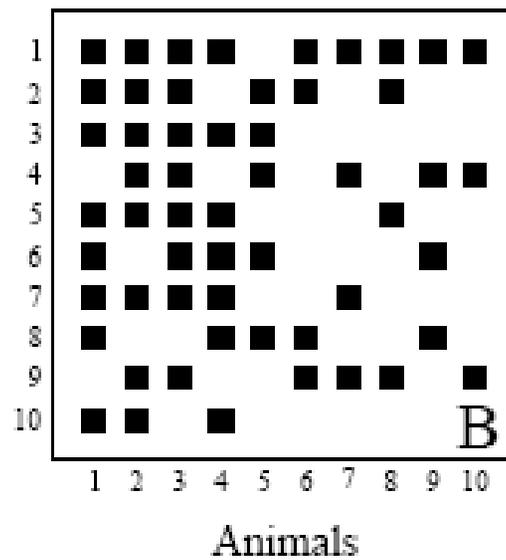
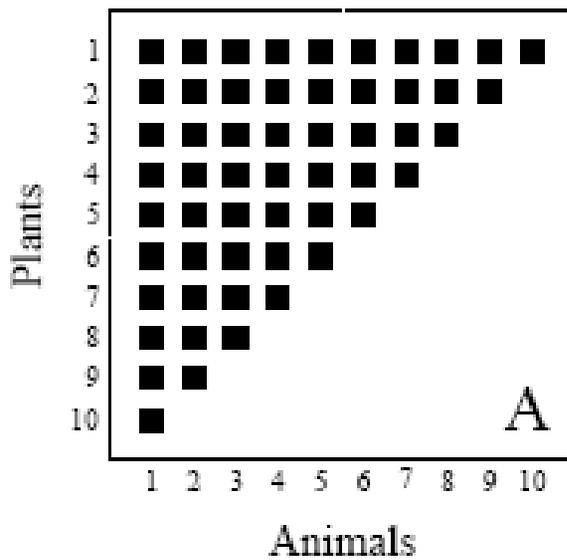
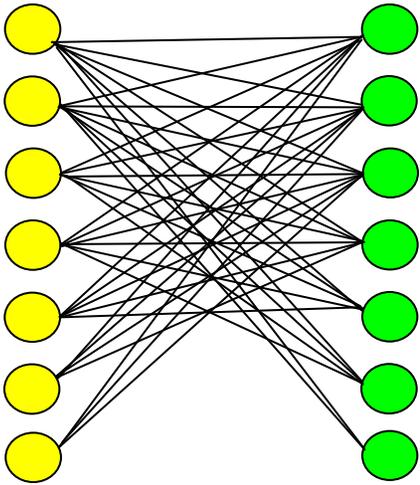


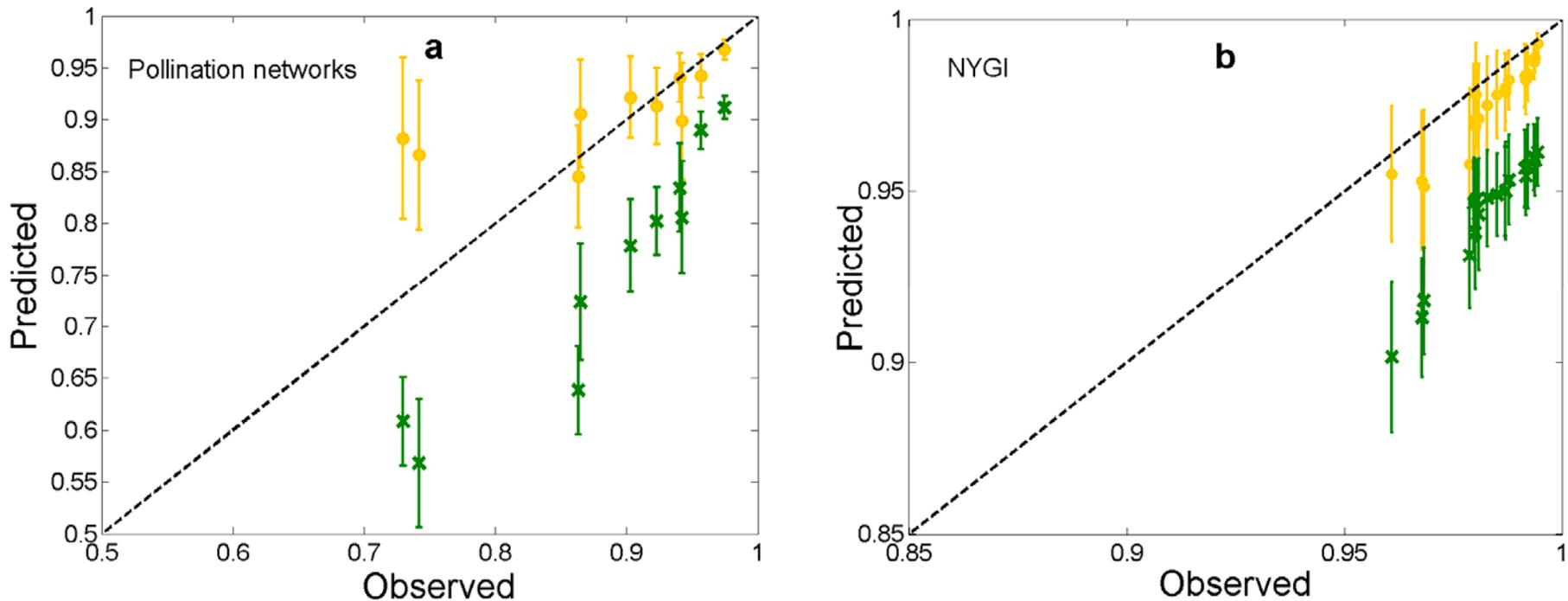
Figure a shows the cumulative degree distribution  $P_{cum}(k)$  for members of class P (plants, manufacturers), and Figure b  $P_{cum}(k)$  for members of class A (animals, contractors). The number of partners  $k$  is scaled by a multiplicative factor of  $1/z_P$  for members of P, and  $1/z_A$  for members of A, where  $z_P=L/P$  and  $z_A=L/A$ . Solid symbols correspond to pollination networks and open symbols to organizational networks. Note that all distributions collapse into a single curve. The solid line corresponds to the model-generated distributions averaged over 1000 simulations.

# Nestedness

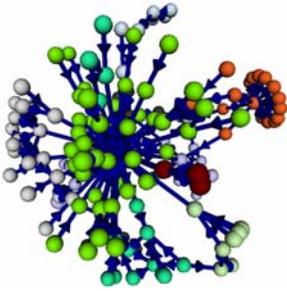


Bascompte et al. *PNAS* **100** (2003)

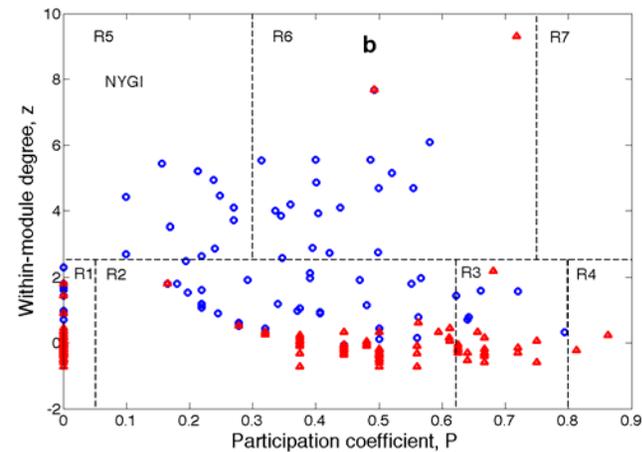
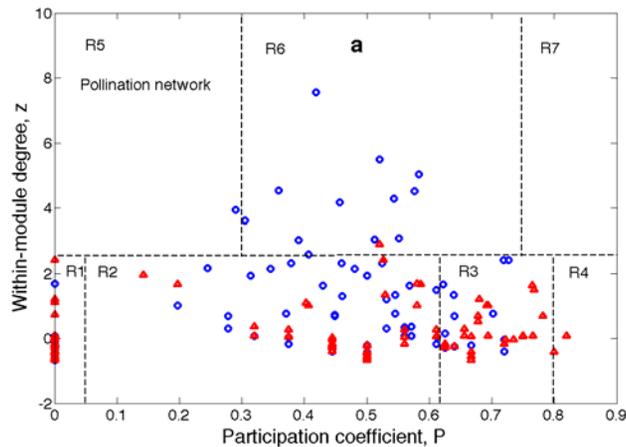
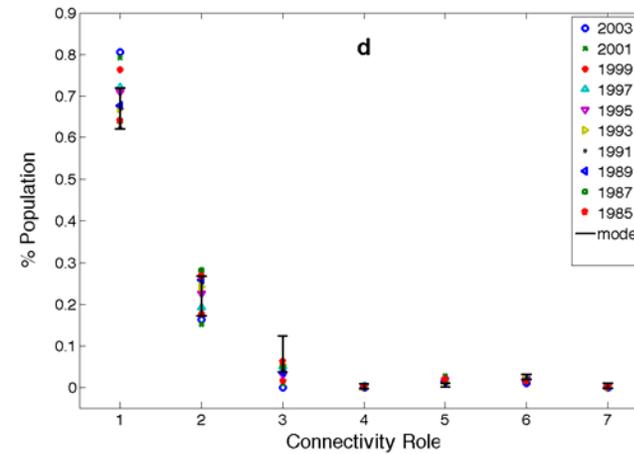
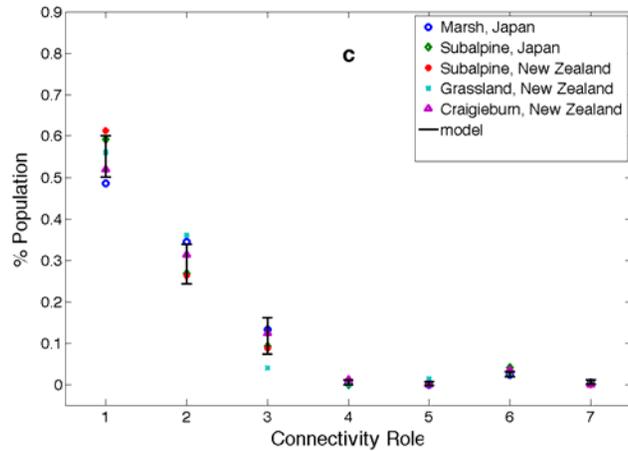
# Nestedness



Nestedness. **a** and **b** show for the NYGI (1985-2003) and the pollination networks respectively, the observed nestedness values (dashed line), and the average plus two standard deviations values generated by the model (yellow dots) and the random assemblages (green crosses) following Bascompte's et al.<sup>25</sup> null model. Here, a nestedness of 0 means a perfectly nested matrix. Note that the two standard deviation bars account for  $-2 < Z\text{-score} < 2$  as defined by  $Z = (\text{observed} - \text{average predicted}) / \text{st. dev. predicted}$ . This analysis was carried out over 1000 simulations.



# Communities & connectivity roles



Connectivity roles. **a** and **b** show for the NYGI (1985-2003) and the modular pollination networks respectively, the percentage of population for the observed connectivity roles (colors), and the values (avg $\pm$ stdv) generated by the model (black bars). We followed Guimera's et.al.<sup>26</sup> classification for nodes, where roles 1-4 are classified as non-hubs and roles 5-7 as hubs.

# Conclusions

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- Simple stochastic model which generates many of the structural features of mutualistic networks
- Unexpected correspondence between:  
*Ecological Networks* ↔ *Organisational networks*
- Other examples where bipartite cooperation model applies?
- Need more systematic approach for assessing goodness of fit.

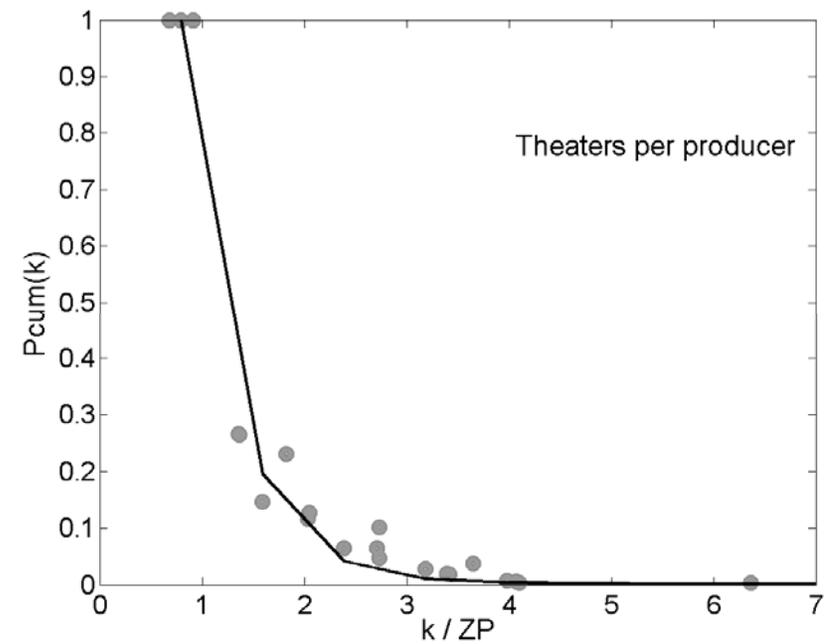
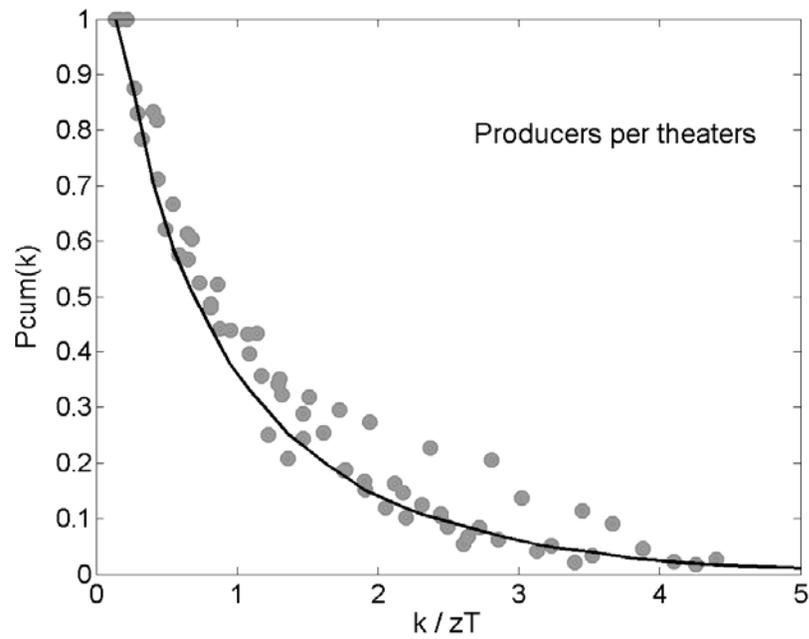
## References:

Serguei Saavedra, Felix Reed-Tsochas and Brian Uzzi (2008), "A simple model of bipartite cooperation for ecological and organizational networks", *Nature*, in press.

Serguei Saavedra, Felix Reed-Tsochas and Brian Uzzi (2008), "Asymmetric disassembly and robustness in declining networks", *Proceedings of the National Academy of Sciences*, in press.

# From fashion to Broadway?

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Thank you

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