Physics and Complexity

David Sherrington
University of Oxford & Santa Fe Institute
Dictionary definition:

Branch of science concerned with the nature and properties of matter and energy

But today I want to use it as much as a mind-set with valuable methodologies
And to show application
to many complex systems in many different arenas
Physics
as sometimes portrayed

Particle Physics
‘Fundamental’ particles

Search for the
‘Theory of everything’

Cosmology
How it all began
But not today

‘More is different’

Particle Physics                              Cosmology
‘Fundamental’ particles                      How it all began
‘Theory of everything’

TOE is by no means the whole story

Many body systems often give new behaviour through co-operation

Both ‘fundamental’ and applicable
Examples of emergent phenomena

- Superconductivity
- Magnetism
- Giant Magnetoresistance
- Quantum Hall Effect
Useful & often give very high accuracy

- Superconductivity
  - Flux quantization
- Magnetism
- Giant Magnetoresistence
  - Basis of modern high capacity data storage
- Quantum Hall Effect
  - Quantized conductivity plateaux

Highest accuracy measurements of fundamental constants even in dirty systems
• Many body systems

• Cooperative behaviour complex
  – non-trivial and new
  – not simply anticipated from microscopics
  – even with simple individual units
  – and simple interaction rules

• But with surprising conceptual similarities between superficially different systems
Typical approach

- Essentials?
  - Minimal models
  - Comparisons/checks: e.g. simulation
  - Analysis: maths & ansätze

- Important consequences?
- Universalities?

Build

- Conceptualization
- Generalization
- Application
Key ingredients

Frustration
Conflicts
Disorder
Frozen
or time-dependent; e.g. uncertainty
Emphasis

- Novel physics
- New concepts
- Minimalist models
- Interdisciplinary transfers
- Much ubiquity, some differences
- Relevance of noise and memory
- Applicability
Examples

Spin glasses

Hard Optimization
Information Science
Computer Science
Mathematical Physics
Biology
Economics
Glassy Materials
Probability Theory
Examples

Spin glasses

Hard Optimization
Information Science
Computer Science
Mathematical Physics
Biology
Economics
Glassy Materials
Probability Theory
Rugged Landscape Paradigm

Two-dimensional cartoon of high dimensional concept

Many metastable states

Hierarchy

Valleys within valleys

Cost
to minimise

Coordinate

Hard to minimise: sticks: glassiness

c.f. motion
Control functions

\[ F(\{J_{ij\ldots k}\}, \{S_{ij\ldots}\}, \{T\}) \]

Statics: Fixed, Variable
Dynamics: Slow, Fast
External influences

General theoretical structure
Control functions, but who controls?

- **Physics**: nature/physical laws
- **Biology**: nature but not necess. equilibrium
- **Hard optimization**: we choose algorithms
- **Information science**: we have choice
- **Markets**: partly supervising bodies, partly manufacturers, partly speculators
- **Society**: governments can change rules
Disordered magnetic alloys, e.g. Au\(_{1-x}\) Fe\(_x\)

- Competitive magnetic interactions
- No periodicity → no simple best compromise

Non-periodic magnetic moment freezing
Slow macrodynamics/ history-dependence/ aging
Similar for site or bond disorder
Phase transitions & preparation-dependence

Susceptibility

Field-cooled

Zero-field cooled

AuFe

non-equilibrium  equilibrium
Minimalist Model

Quenched random interaction: ±

Cost or Hamiltonian

$H = -\sum_{ij} J_{ij} S_i S_j$

Magnetic elements

$s = \pm 1$

Spin up/down

Frustration & Disorder
Minimalist Model

\[ H = - \sum_{ij} J_{ij} S_i S_j \]

Simulations ~ experiment

Range-free case

soluble but very subtle
“The Dean’s Problem”

Allocate $N$ students to 2 residences with maximum happiness

Satisfaction
To maximise

$$H = + \sum_{ij} J_{ij} s_i s_j$$

Inter-student friendship: $\pm$

Also $\sum_i s_i = 0$

Dorm A/B

$s = \pm 1$
Phase diagram

Temperature/noise/uncertainty/Dean’s impatience

Easy to equilirate

Hard to equilirate

Many metastable states
‘Rugged landscape Slow dynamics

No freezing

Ferromagnetic freezing

GFM

Attractive bias
Examples

Spin glasses

Hard Optimization
Information Science
Computer Science
Mathematical Physics
Probability Theory
Biology
Economics
Glassy Materials
Examples

• Minimizing a cost
  – *e.g.* distribution of tasks, partitioning

• Satisfiability
  – Simultaneous satisfaction of ‘clauses’

• Error correcting codes
  – Capacity and accuracy
Two issues

• What is achievable?
  – Analogue: “statics”/equilibrium
    • May be hard to find?
    • Is it possible?

• If achievable, how to achieve it?
  – Needs algorithms = dynamics
    • We may be able to devise
    • But glassiness can badly hinder efficacy
K-satisfiability

Simultaneous satisfiability of many ‘clauses’ of length K

\[(x_1 \text{ or } x_2 \text{ or } \overline{x_3}) \text{ and } (x_3 \text{ or } x_4 \text{ or } x_5) \text{ and } \ldots\]

\[\alpha \equiv \frac{M}{N} = \left\{ \frac{\# \text{ of clauses}}{\# \text{ of variables}} \right\}\]

Phase transition(\(\alpha\)): SAT / UNSAT
Compare: K-satisfiability

Physicists recognised this subtlety through comparison with $K$-spin glass.
Potts or $K (>2)$ -spin glass

Where the idea came from

Originally looked at as a purely intellectually interesting extension
Similarly: error-correcting codes

Redundancy

Shannon limit

0

RETRIEVABLE

HARD TO RETRIEVE

UNRETRIEVABLE

Normal algorithms stick

And now we know why
In fact, more regimes

Clustering: Random K-SAT

EASY | HARD | SAT | UNSAT

α*

α_d

α_c

α_s

α
New algorithms

- Understanding brings opportunities
- Normal physics
  - Algorithms given
- Artificial systems
  - We can design algorithms
    - *e.g.* Computational
      - Simulated annealing
      - Simulated tempering
      - Clustering……..

Great advance: Survey propagation
Simulated annealing
effective stat. mech./thermodynamics

\[ Z = \sum \exp(-\text{Cost} / kT_{\text{anneal}}) \]

Artificial ‘temperature’ \( T_{\text{anneal}} \)

Min Cost = \( \lim_{T_A \to 0} T_A \ln Z \)

Optimum achievable

Achieving it requires (algorithmic) dynamics

Frustration & disorder → glassiness

But we can choose the dynamics
Landscape paradigm for hard optimization

Cost

obstacles

Steepest descent gets stuck
Simulated annealing

Probabilistic hill-climbing
Add ‘temperature’: freedom

\[ P(\text{move}) \sim \exp(-\Delta C / T_A) \]
Simulated annealing

Gradually reduce $T_A$

Cost

Variables

Annealing temperature

Variables

Cost

Gradually reduce $T_A$
Gradually reduce $T_A$
Simulated annealing

Hopefully

Good basic tool but now better ones
Examples

Spin glasses

Hard Optimization
Information Science
Computer Science
Mathematical Physics

Probability Theory
Glassy Materials
Economics
Biology
‘Statistical physics of the brain’
Typical neuron

Schematize

(a)

(b)
Schematic neural network

Input

Output
Mathematical modelling

- Neuronal activity: $V_i$
- Synaptic weights: $J_{ij} > 0$ switch-on, $< 0$ switch-off
- Total input: $U_i = \sum_j J_{ij} V_j$
Consequence of input ‘potential’

Output activity of neuron/ probability of firing

- and so on through the network

Input potential

Rounding ~ “temperature” $T$
Maps to analogue of spin glass

\[ H = - \sum_{ij} J_{ij} S_i S_j \; ; \; J_{ij} = \sum_{\mu} \xi_{i\mu} \xi_{j\mu} \]

Quasi-random +/- but trained
Attractors: tuned metastable states

- Associative memory
  - ‘attractors’
  - ‘basins of attraction’
  - memorized patterns
determined by \( \{ J_{ij} \} \)

- Many memories
  - many attractors
  - require frustration

Phase space
Rugged landscape analogy

Valleys $\sim$ attractors \quad Sculpture $\sim$ learning

$\{s_i\}$ \quad $\{J_{ij}\}$

Different timescales

fast retrieval \quad slow learning
Phase diagram: Hopfield model

Synaptic ‘temperature’

Para
(No attractors)

‘Spin glass’
(metastable attractors unrelated to memories)

Retrieval

Capacity: Pattern interference noise

c.f. ferro
Extensions

- **Artificial neural networks**
  - We design
    - Non-biological elements
    - Train by experience

- **Other biological evolution**
  - self-train/select
    - maybe without knowing what is “good”
    - e.g. evolution of proteins from heteropolymeric soup
    - Autocatalytic sets
Spin glasses

Examples

Hard Optimization
Information Science
Computer Science
Mathematical Physics
Probability Theory
Biology
Economics
Glassy Materials
The Stockmarket

- **Price** vs. **Time**

1. **Different strategies** (Disorder)
2. **Buy & sell** (Dynamics I)
3. **Learn from Experience** (Dynamics II)
4. **Common information** (Mean field)

*Not all can win* (Frustration)
Minority game

N agents 2 choices
Aim to be in minority

Individual strategies → Collective consequence
• act on common information (e.g. minority choice for last m steps)
• preferences modified by experience (keep point-score)

Correlated behaviour & phase transition
Phase transition & ergodicity-breaking

Random strategies, random histories

Phase transition: $\alpha_c$

minimum in volatility

$\alpha < \alpha_c$ non-ergodic

$\alpha > \alpha_c$ ergodic

c.f. spin glass susc.
Coarse-grained time-average

Effective interaction between agents

\[ H = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i \]

Quasi-random \( J \) and \( h \) related to agent strategies

c.f. spin glass or neural network

Strategy point-score dynamics for agents with 2 strategies

\[ p_i(t+1) = p_i(t) - \partial H / \partial s_i \bigg|_{s_i = \text{sgn} p_i(t)} \]
Minority game

Phase space

\[ H = + \sum_{ij} J_{ij} S_i S_j \]

\[ J_{ij} = \sum_{\mu} \xi^\mu_i \xi^\mu_j \]

Many repellors
Macro dynamics

Generating functional
Map to macroscopic variables (multi-time)

\[ p(t+1) = p(t) - \alpha \sum_{t' \leq t} (1 + G)^{-1}_{n'} \text{sgn} p(t') + \sqrt{\alpha} \eta(t) \]

"Representative agent ensemble"
Simulations & iterated theory

Initial bias

\( p_i(0) = 0 \)
\( p_i(0) = 0.5 \)
\( p_i(0) = 1 \)

Open = simulations    Solid = numerical iteration of analytic effective agent equations
**Limit-order book**

Agents place or remove orders: buy, sell, market. May be executed. Speculators gain on price changes. Manufacturers must absorb → liquidity.

But how do they choose what to do? Evolution of strategies?

Driven by individual attitudes, co-operative actions, learning?

c.f. Evaporation-deposition-annihilation
Spin glasses

Examples

Glassy Materials
Hard Optimization
Information Science
Mathematical Physics

Economics
Biology
Computer Science
Probability Theory
Symbiosis of techniques

- Theoretical physics interplay
  - Minimalist modelling
  - Sophisticated mathematical analysis
  - Computer simulation
    - Both to check with more complicated real world
    - And to do experiments for which no real analogue
  - Conceptualization
- Real experiment
Useful interdisciplinary transfer through physics

Not only of materials and experimental methods but also of concepts & mathematical techniques for Understanding, quantification & application

And there are many more applications still to consider
Caveats

• I have only given brief indications
  – Needs much fleshing
  – but I hope illustrative of possibilities

• Concentrated on macroscopic properties
  – Not individuals

• And on typical/average behaviour, not fluctuations
  – e.g. Not a guide for stockmarket speculation

• But one could do more
  – And there is much more to do
Collaborators
Teachers, colleagues, students, postdocs, friends

Tomaso Aste
Jay Banavar
Ludovic Berthier
Stefan Boettcher
Arnaud Buhot
Andrea Cavagna
Premla Chandra
Tuck Choy
Ton Coolen
Dinah Cragg
Lexie Davison
Andrea De Martino
Malcolm Dunlop
Alex Ducring
David Elderfield

Julio Fernandez
Tobias Galla
Juan Pedro Garrahan
S.K.Ghatak
Irene Giardina
Peter Gillin
Paul Goldbart
Lev Ioffe
Robert Jack
Alexandre Lefèvre

Phil Anderson
Sam Edwards
Walter Kohhn

Turab Lookman
Peter Kahn
Scott Kirkpatrick
Helmut Katzgraber
Stephen Laughton
Francesco Mancini
Marc Mezard
Esteban Moro
Peter Mottishaw
Normand Mousseau
Hidetoshi Nishimori
Fernando Nobre
Dominic O’Kane

Reinhold Oppermann
Giorgio Parisi
Richard Penney
Albrecht Rau
Avadh Saxena
Manuel Schmidt
Hans-Juergen Sommers
Nicolas Sourlas
Byron Southern
Mike Thorpe
Tim Watkin
Andreas Wendemuth
Werner Wiethege
Stephen Whitelam
Peter Wolynes
Michael Wong
Theoretical methodology

- **Statics/thermodynamics:**
  - Partition function

\[
Z = Tr\{\exp[-\beta H]\}
\]

- **Dynamics:**
  - Generating functional

\[
Z = \int DS(t)\delta(\text{microscopic eqn. of motion})
\]

* Transform to macrovariables: average over disorder
  - Multi-replica/ multi-time correlation & response fns

* Infinite-range
  - extremal dominance ~ solubility + subtlety)